The story of how technology

can make learning real and relevant,
create more effective school systems,
and double college graduation rates
at half the cost.

Make It Real

Art Bardige

On

The Future of Education

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Draft 4.1.19

What if Math, www.whatifmath.org, is a project of Sustainablearning a 501(c)(3). Available free to students and teachers, its hundred+ Labs (Problem-Based-Learning lessons) built on spreadsheets as math laboratories to ask "What if…"

Join me on the book/course website makelearningreal.org stretches these ideas.

My biography as well as the Pattern of Knowledge can be found on my personal website www.artifacts.com.

Cover Image – TBD

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For: All of Our Kids

This Abstract, which I now publish, must necessarily be imperfect. I cannot here give references and authorities for my several statements; and I must trust to the reader reposing some confidence in my accuracy. No doubt errors will have crept in, though I hope I have always been cautious in trusting to good authorities alone. I can here give only the general conclusions at which I have arrived, with a few facts in illustration, but which, I hope, in most cases will suffice. No one can feel more sensible than I do of the necessity of hereafter publishing in detail all the facts, with references, on which my conclusions have been grounded; and I hope in a future work to do this.

Charles Darwin*, On the Origin of Species,* 1859

"The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.

Hermann Minkowski*, Space and Time, Lecture* 1908

Prologue. Hope

I hope this work will lead you to question some of your most cherished and fundamental beliefs about education. I hope it will give you a new optimism that education can and certainly will change in profound ways to bring creativity and choice to lifelong learning for every one of our kids. And I hope it will provide you with examples you can build both your new vision and the future of schooling on.

It will, I hope, make the old foundation stones of schooling, reading, writing, and calculating, "fade away into mere shadows[1]," for they are barriers blocking all too many of our kids from learning the kinds of advanced skills they will need. The old skills limited communication, encouraged an emphasis on mechanical paper processes, and make collaboration difficult. We can now build schooling on a new base, a foundation "sprung from the soil of…" 21st century jobs, designed and constructed for the digital age, using the digital tools our kids must master.

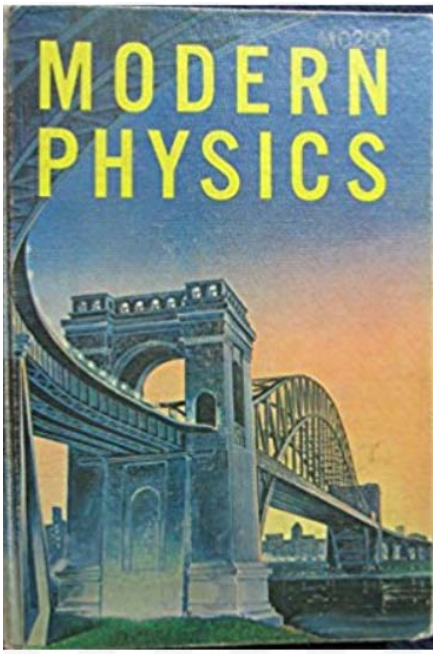
It will, I hope, make the distinction between teaching and learning, instruction and curriculum, fade away as well. Thus far, digital technology has been, in the main, used to change methods of instruction. When applied to curriculum, it has not questioned the relevance of the traditional content; instead it has replicated the paper lessons. ***Until we change what we learn, we cannot use technology to change how we learn,*** *for "only a kind of union of the two will preserve an independent reality."*

It will, I hope, encourage you to believe in the opportunities for the tools of the digital age to reinvent schools and change education. We have become cynical about education, lowering expectations, harshly judging our schools, our teachers, and even our kids. We have tragically come to even question a fundamental tenet of our nation that education can and will give our kids a better life. The technology of the digital age has the capability to change that; the capability to enable every student who wants to, to get a rich and rewarding college education at a price that is affordable. We can meet our great educational challenge to: double our college graduation rates at half the cost well within a generation by empowering schools, teachers, and students with digital age technology and the freedom to use it openly. We can decide that schooling should be about the skills our kids will need to thrive and not about the content they need to remember, about learning and not instruction, about making students teachers and teachers students, about the future and not the past.

[1] Minkowski, "Space and Time" Lecture 1908

1. “Lord Knows it needs Something.”

Mady[2] blurted out in her naturally poetic voice to address the challenge, “How do we fix education?” Her words echo universally, our educational system needs to change.



My 1959 high school physics book by Dull, Metcalfe & Brooks

I began to dream of answering that question while studying to be a teacher. I planned to write a physics textbook that would enable every student to love and learn my favorite subject. You see, in 1958, I was taught physics from this textbook[3]. Neither modern nor physics, its author’s name describing its content, its cover defining its attitude, it sufficed until then because the digital technology revolution was just starting. Like the rest of education, though dull, boring, and often ineffective, its 19th century industrial age design and paradigm had produced the kinds workers business called for.

Sixty years later our world has profoundly changed. It has become digital. Our schools have not. Sure, they make use of digital age technology for automating many of their functions, but the underlying paradigm has not changed. We retain an obsolete curriculum, mired in paper-based goals and a pedagogy focused on teachers being the source of knowledge and skills. The digital age requires us to rethink both curriculum and instruction, to reinvent education. The demands of our digital age have created the problems we face in education. Digital age technology has to solve them.

This is the story I will tell you. It is the story of discovery, invention, and hope. It is the story of all the human technology revolutions that make us more effective, efficient, and relevant. And it is the story of reimagining education for all our kids to thrive in the digital age by using real digital tools to do real problem solving in schools open to the real world. By making education in the digital age real, we can meet the audacious necessary challenge of our age, to double college graduation rates at half the cost.

[2] Mady Holzer, educator, poet, and friend, in a private conversation
[3] Dull, Brooks, Metcalf, Modern Physics, Holt 1955

An Unlikely Revolutionary

I was reading a small biography[4] of an obscure medieval mathematician, Leonardo of Pisa[5], for fun and pleasure when I made the surprising discovery that profoundly changed my vision of the future of education and sent my life and work careening in new directions.

Leonardo of Pisa (c.1170-1250)

Leonardo was born in Pisa around 1170, the year the foundation stones to the infamous Leaning Tower were laid. Shipped off to Algeria as a young boy to join his father, a trader and agent for Pisan merchants, this precocious boy was tutored in Arabic arithmetic and algebra, then academic subjects not used by business in either the Arab or European worlds. Following in his father's footsteps, Leonardo became a trader.

Before I introduce you to my exciting and surprising discovery, I think it only right that I introduce myself so that you can understand why it is so important to me. I grew up loving technology and innovation. After getting a bachelor's degree in physics and then a master's in teaching at the University of Chicago where I also learned to love the liberal arts, I taught two different innovative programs in high school physics where student labs and film played a major role. I did a stint as an educational filmmaker in science and math that taught me to visualize and use words sparingly to caption silent images. I followed by teaching junior high math writing my own curriculum which brought me to a much wider educational world, soon expanding as math coordinator across the breadth of math PK-16[6]. Technology became central again in early 1978 when I convinced my forbearing wife that I had to have one of those new Apple computers. Two years later, believing this was the way for me to express my vision of education, I became a tech entrepreneur, designing programs, learning business, and building the first of 3 companies to develop technology for learning. I sold the first company and worked with and at Simon and Schuster as a chief scientist to plan and design tech products across their wide educational sphere from early childhood to executive business training. My latest venture, a non-profit,[7] will

The House of Wisdom c.800

[4] *The Man of Numbers, Keith Devlin,* Bloomsbury, 2011
[5] You may have heard of him as Fibonacci, a name given him some 600 years later.
[6] PK-16 is the shorthand for prekindergarten through college.
[7] Sustainablearning

be described in detail later because it is the result of this discovery. It shares my time with my work as a trustee at Lesley University, a great education and design school.

Leonardo was a 30-some-year-old trader in the year 1200 when he returned to his native Pisa to bring the Arabic arithmetic and algebra, he had studied to solve the problems of Pisan businesses. For you see, the math used by medieval merchants in both the European and Islamic world, was Roman, and the calculation technology an abacus. Good enough for the Empire with its common currency and weights & measures, it handicapped Pisan merchants who were constantly converting the variety of measures and coinage of the medieval world, solving ratio-rich problems that required complex multiplications and divisions rather than simple additions and subtractions. The book Leonardo published two years later, *Liber abbaci* (*The Book of Calculation*), applied Arabic math to the needs of medieval business.

"None of the cities should be allowed to have its own separate coinage or a system of weights and measures; they should all be required to use ours."

Dio Cassius 235 AD

Romans multiplied by doubling and adding on an abacus.

Keith Devlin's biography[8] focused on the dissemination of Leonardo's vision across medieval business. To make his points about Leonardo's critical role in developing schools to teach his new methods for math in business, he included an image of *Liber abbaci's* table of contents. The page left me spitting out expletives. I saw a near replica of today's K-12 math curriculum. You may not at find it such, for the chapter headings in medieval manuscripts were less overviews and more initial sentences. But I had the good fortune of having had a great deal of experience with the K-12 curriculum scope and sequence from my work as a

1. *On the recognition of the nine Indian figures and how all numbers are written with them. (place value)*
2. *On the multiplication of whole numbers*
3. *On the addition of them, one to another*
4. *On the subtraction of lesser numbers from greater numbers*
5. *On the division of integral numbers*
6. *On the multiplication of integral numbers with fractions*
7. *On the addition and subtraction and division of numbers and fractions and the reduction of parts to a single part*
8. *On the buying and selling of commercial things (ratio & proportion)*
9. *On the barter of commercial things (rate)*
10. *On companies made among parties (percents)*
11. *On the alloying of money (mixture problems)*
12. *On the solutions of many problems (Fibonacci sequence)*
13. *On the rule of elchataym by which problems of false position are solved. (solving linear equations)*
14. *On the finding of square and cube roots, on binomials and their roots.*
15. *On the pertinent rules of geometric proportions*

Leonardo's Math today

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[8] Keith Devlin, *The Man of Numbers* 2011, Bloomsbury Press

teacher, math curriculum coordinator, and math digital content developer long fascinated by the scope and sequence seeking to order it in more rational ways. Though, I like everyone else, thought this sequence immutable, basic and fundamental, I recognized these chapter titles as parts of a whole! And soon filled in most of the synonyms by reading Leonardo's text.

Leonardo's Math

Arithmetica favors the algorist over the abacist (woodcut 1504)

I realized that the math our kids are required to master in the 21st century is neither basic nor fundamental. It was not the "natural" math sequence. Our curriculum was designed by Leonardo of Pisa for medieval business in the year 1202 from concepts developed by Arab scholars.[9] It used paper, the technology introduced to Europe a century before, as the tool for algorithmic calculation replacing the abacus. Without paper, Leonardo's algorithms and algorithmic calculations with symbols would not have been easy or in many cases possible. This technology and methodology would prove so powerful that over the next 4 centuries Leonardo's math slowly became fully symbolic and this new "technology" replaced Roman math as the standard system for calculation.[10] It changed little over the following 400 years.

Today, business no longer calculates on paper. It no longer uses Leonardo math. Since the invention of the digital spreadsheet in 1979, it has used the tools of digital technology, digital instead of paper algorithms, discrete instead of continuous variables, and functions instead of equations, to solve problems. It asks, "What if…" not "What is___?" We are educating our kids for the 13th century and when they will need the quantitative tools of the 21st.

$\underline{5280 + 1732 =}$ $\quad \begin{array}{r} 647 \\ \underline{*44} \end{array}$ $\quad 6\frac{2}{3} - \frac{1}{8} =$ $\quad \frac{5}{6} / \frac{-7}{12} =$

$\frac{438}{25} = 17\, r14$ $\quad \sqrt[3]{64} + \sqrt{81}$ $\quad a^2 + b^2 = c^2$

$A = \pi r^2$ $\quad 3x - 7 = 11$ $\quad \frac{16}{9} = \frac{6}{x}$ $\quad \frac{10}{7}x + 1 = \frac{3}{2}x - 8$

$2x^2 - 8x + 14$ $\quad (15x^2 + 8x - 4)/(3x + 1)$

$\frac{-x}{x^2 - 6x + 5} + \frac{-x - 1}{x^2 - 10x + 25}$ $\quad \frac{4}{6\sqrt{3}}$

$\sqrt[3]{6x - 4} = \sqrt[3]{5x + 8}$ $\quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Paper-based exercises

Why are we teaching our kids medieval math when we want them to learn how to solve digital age problems?[11] What if we reinvent mathematics education for the digital age? Would it include paper algorithms like subtraction with borrowing,

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[9] Devlin makes the argument in The Man of Numbers that Leonardo's work slowly spread across Europe and became the basis for the standard Medieval math curriculum.
[10] This 1508 lithograph depicts the goddess Arithmetica choosing the algorist and not the abacist.
[11] Some may argue that this ancient math builds understanding. We will soon find that it does not.

3-digit multiplication, or long division? Would it include operations on fractions or practice conversion of units? Would it include the algebra of solving equations for unknowns, invented by al Khwarizmi 1200 years ago for solving equations? Would it include the quadratic formula? When every student carries a powerful computer in their pocket that handles all these calculations instantly and with ease, why do any students have to learn to do these calculations, “the old way” by hand?

From the Common Core Website

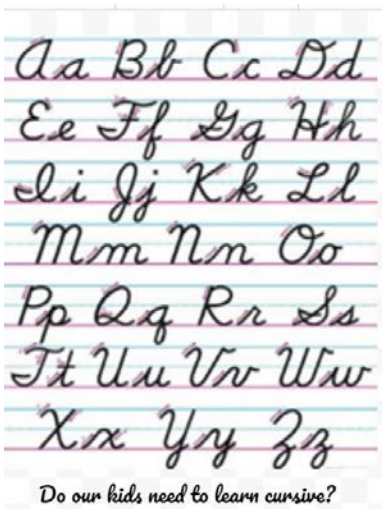
Once we take out the paper algorithms not only does the traditional math curriculum collapse into a much smaller ball, its order, the scope and sequence, becomes irrelevant. For the concepts (add, subtract, multiply, divide) are all the same level of abstraction, division is no more difficult than addition, multiplication actually easier than subtraction. It was the difficulty of the algorithms that defined the standard school progression. Take out the myriad of worksheets our kids repetitively do to practice these paper algorithm skills and we are left to reinvent most of the math we want them to learn.

Now you may say, wait a minute! Without this practice how are our kids to learn the concepts? You may even believe as many business executives in my experience do, that the reason their younger colleagues don’t “get” math and can’t solve problems in their heads, is that schools have been permissive about calculators and fail to demand real paper practice as they used to. The problem they see is real but not the cause. I ask, “Where is the concept of division in long division?” Paper algorithms were developed to speed calculation in medieval counting houses, they have little if anything to do with understanding the concepts or applying them to the solving of problems. Though the multiplication algorithm depends upon the distributive property, few teachers, let alone students, understand that. And the first stumblestone for many, borrowing/regrouping to subtract had nothing to do with the concept of subtraction.[12]

Though I’ve spent most of my working life focused on developing technology to change the way kids learn, this revelation about Leonardo’s math led me to understand that **technology changes not just the way schools transfer knowledge, but the knowledge they must transfer**. I realized that technology is used in education today to automate student practice and teacher presentation. It has, thus far, not changed what we expect of students, or what we believe they need to learn. It is to schools what a modern assembly line would be to building horse and buggies. After you’re done, you’re still a driving horse and buggy. You haven’t solved your transportation problems**.** In schools today, technology does not make student practice relevant or teacher presentation more effective.

[12] This picture of the Common Core staircase is taken from their website.

“That would be crazy!”

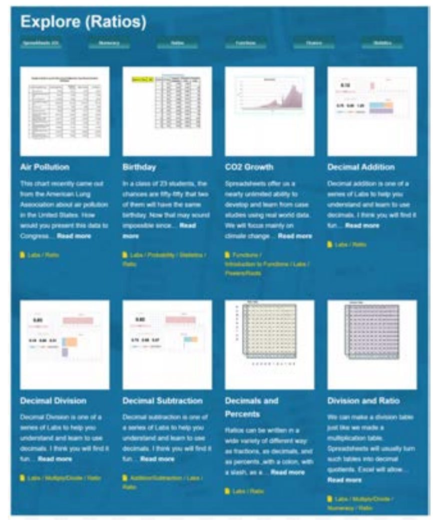


Walk the halls of most of the elementary schools in America today and you still see bulletin boards and walls decorated with manuscripts, student writing about a visit, a person, or some other topic. Many are decorated with pretty images, the text handwritten in each student’s best penmanship. They could be monks decorating the halls of their monasteries in medieval Europe. That is the reality in schools today despite a very different reality in business and industry.

“Why would anybody write without a computer today?” asks my wife Betty, a professional writer, professional educator, and the last person in the world who you would expect to be an early adopter of any technology. I am usually pleasantly surprised when she knows how to turn on the lights in the house. In 2 minutes, she rattled off a long list of the things that Microsoft Word enables her and any writer to do: cut and paste, look up something, copy it to paraphrase it, spell check, get a quick outline, highlight things to do them later, reorganize a paper, search for redundancies, look up a word, get immediate research, of course edit and correct anything, add graphics, keep a word count, or change the font, titles, and layout to make it beautiful. You can, if you want, share not only drafts and finished versions, you can share the writing in today’s word processors. Even if we were to follow today’s pre-Word curriculum, computers change the process and increase efficiency. They make it much easier to write and to rewrite.

Some may try to argue that handwriting allows us to think about what you are writing, that the physical process of drawing the letters and moving our hand across the paper builds a connection with the brain. I doubt it. If that were true, typewriter proficiency could separate good writers, creative thinkers, from hackers. When I moved from writing with pen on paper to writing on typewriters, the only thing I lost were my cramped hand and that hard callus on my middle finger. I went from daily writing 2 to 3 pages on a legal pad to 6 or 8 pages that were legible. When I moved from typewriters to computers, I gained all the things that Betty mentioned and more, losing only the continuous humming of the electric typewriter and the Wite-Out I kept next to my typewriter to correct mistakes. My productivity went from half a dozen pages a day to nearly a dozen on a good day. I believe that the quality of my writing also improved. I was doing more of it, with a much better flow, because the physical process of putting words on the page was sped up.

If mechanical efficiency were the only things that made computers the writing tools, we should be teaching our kids to use, it would be worth it to teach students to use word processors. It would at least be a start in preparing them for their 21st century jobs. But mechanical efficiency, while reason enough to put a computer on every student desk, is only the beginning. For today, I like to think not in terms of writing but in terms of communication, and when I think in terms of the broad vision of communication I think about WordPress. For those of you who have not heard about this amazing program, it is the standard tool across the world for blogging and developing web pages. It is free, maintained as an open-source platform that handles a quarter of the websites and half of the total database space on the Internet. It has two essential functions. It lets people build and maintain a blog and develop and enrich web pages[13]. I think of WordPress as a 21st century communication and collaboration platform. With it, I can build a website that includes text, images, links, galleries of images, video, along with a host of specialized “plugins” that enable me to add surveys, enable registrations, collect money, get email addresses, build timelines, explore databases, calculate, and even play games; all the things we have gotten used to websites doing.



Explore Menu on whatifmath.com -- as posts

Blogs are new to the 21st century and surprisingly powerful ways of communicating. They let us post ideas of all kinds in every configuration we can imagine. And they let others comment and elaborate on those posts. In *What if Math*[14]*,* we use this idea of posts to create not only our blog but to put up all our Explore[15] content, our Labs[16]. As we develop new lessons, we post them, treating curriculum as if it were a giant, ever growing and ever-changing blog. Students and teachers can open these lessons as if they were a blogpost or download them to use as an application on their computer, perform them, comment on them, link them with other Labs, and even grade them. WordPress lets us easily show our blogpost Labs as a picture gallery and to categorize our lessons in a wide variety of ways. WordPress not only puts these posts up on our Website, it sends out an email to our mailing list announcing and describing them and tweets to our Twitter account to notify our followers, and it will do the same for any social media account we have including Facebook, Pinterest and more.

“Writing” in the digital age is now an interactive, sharing, linked, visual communication tool that our children already use. Our job is to help students learn to use digital media tools like WordPress to become effective, efficient, and relevant communicators in their 21st century careers. The reason Internet companies are today the richest and most profitable companies in the world is that they are designed to enable their users to be more effective, efficient, and relevant communicators. And we have only just begun to really explore this new, only a decade old, amazing tool, the printing press of our new world.

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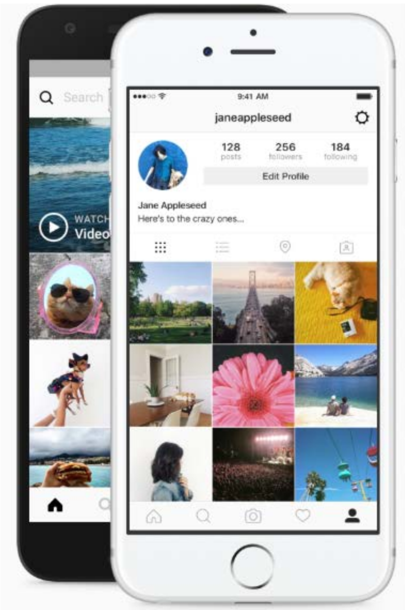
[13] Our whatifmath.org website as well as my personal artifacts.com website are built and maintained on WordPress.
[14] whatifmath.org is our vision of mathematics education of the future. It is free, student focused, and problem-solving based.
[15] Students can choose the lessons they want in the order they want because we want them to be explorers.
[16] We call our lessons Labs because we treat lessons as student laboratory experiences.

Instagram



Today, text captions pictures

What does a world class photographer do today to earn a living? My friend Richard Sobol is a world class photographer with a lifetime of experience in photographing everything from baby seal hunts to Egyptian-Israeli peace treaty signings. He has been, I venture to guess, to nearly every part of the world and has photographed their inhabitants, their buildings, and their lives. He is a great photographer, one of the best, and yet he can no longer make money in photography now that everyone who uses a phone can take great pictures and can travel relatively cheaply to exotic places. Many of us take more pictures on our cell phones every year than Mathew Brady[17], the great Civil War photographer took during the war.

Yesterday as we sat in our favorite sidewalk cafe in the exotic City of Cambridge, he told me the story of his once again reinvention that would enable him to earn a living. For today he has taken to writing books, mainly for children, about photographing exotic animals and going to schools to talk about that writing and encourage students to tell stories visually and verbally. He told me the story of a friend, another world class photographer who has taken to selling real estate to supplement his meager photo income. Discussing ways to reach out to schools and magazine editors to sell their photos or photo ideas, his friend told him he had to use Instagram! Not on his radar screen; Richard had been spending his time, energy, and money on his website and email campaign, now he is on the search for help up the Instagram learning curve.

http://www.richardsobol.com/

In my eyes, this is not a story of social media or technology change; those stories are familiar. **This is the story of a revolution in the way we communicate, the way we transfer information, the way we learn, and most of all *what* we learn**. Embedded in these stories is the significance that images gained in the 20th century, and a new kind of picture power has begun to even replace text in the 21st century. We have long recognized that

[17] Mathew Brady along with his crews produced more than 10,000 plates of the War. Every **two** minutes, humans take more photos than existed in total **150** years ago.

“a picture is worth a thousand words” and that a great picture captures an emotion and can tell an important story all by itself. What we had not recognized before is that pictures, both still and moving, are becoming our primary storytelling medium.

I like to describe the change as:

Yesterday – pictures captioned text. Today – text captions pictures.

Yet, in our schools, text remains our primary means of communication and primary focus as we keep working to increase the seat-time given to teaching reading and writing. Our arts are given, if our students are very lucky, a half hour a week with a dedicated art teacher, while studying reading and writing on paper more than an hour and a half a day. And those few minutes a week are likely dedicated to art not design, to making pictures not using them, to using existing skills not learning new conceptual ones.[18]

A Broken Business Model[19]

The West Gate at the oldest college in America, Harvard University. Even Harvard is concerned about its rankings.

The question I am asked most about college education is: “**Why does it cost so much?**”

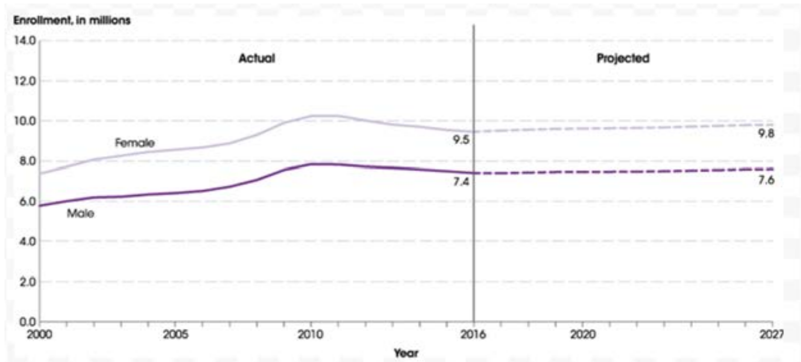
The inflationary period began in earnest in the 1980’s spurred on by the *US News and World Report* college rankings. In 1983, in an effort to compete with its much bigger rivals Time and Newsweek, US News ventured into college rankings. Under Marvin L. Stone, it surveyed “1,308 college presidents to get their opinions on which schools offered the best education.” They repeated that “academic-reputation-only method” through 1987. Beginning in 1988 to be more objective and tech focused, they started using statistical data as part of the ranking methodology, evaluating those numbers along with the results of the survey.”[20]

[18] Steve Bayle describes Richard’s transformations as “Richard realized he was in the communications business, not the photography business. Just like the railroads should have realized they were in the transportation business, not the train business. People confuse and conflate their tools with the business they are in.”

[19] Michael Horn, “Will Half of all Colleges Really Close in the Next Decade?”

[20] All these quotes from https://www.usnews.com/news/national/articles/2008/05/16/the-birth-of-college-rankings

To ensure that this subtle data shift maintained the reliability of the rankings, the editors did not include costs, notably tuition in the data, for this would have substantially altered those rankings causing schools like the Ivy's with high tuition rates to fall down the list. As college presidents and trustees recognized that this college rating system was becoming more and more popular, going online in 1997, they pushed their institutions to pay attention to the elements that made up the rankings. They opened the institutions pocketbook to pay for higher faculty salaries, smaller class sizes and student-faculty ratios, proportion of full-time faculty, along with student rankings like SAT scores and graduation rates to attract higher "quality students". **The result, described by many as an arms race, has continued unabated to this day.**



College enrollment has level out and is expected to shrink

Instead of increasing productivity and thus decreasing costs, colleges and universities at all levels have decreased productivity and increase costs. They have already reached a point in this feeding frenzy that is widely recognized as unsustainable. **Simply put, our customers cannot afford our product.** Our kids cannot afford to get a college degree and are burdened just as they are starting out in their adult life with an unmanageable.[21] College enrollments have been decreasing and are projected to be flatlined or even to drop over the next decade.[22] Our colleges, stressed already, will be under increasing competitive and financial pressure.

Any business looking at these numbers and trends would be looking for a new business model, for this one is unsustainable. Colleges know they are facing increasing competition from Web-based educational opportunities. They know their international students, who they have been relying on to pay full freight tuition, will be finding alternatives in their home or other countries as the 'flat world" rivals move into our competitive space. They know they are increasingly discounting their tuition and increasing their budget deficits. And they know their costs are rising as they continue to try to compete on the old model, for they do not have to look far to see the amazing successes of Southern New Hampshire University or Arizona State University.

This broken business model is not only a higher education problem, it is a K-12 problem as well, where suburban school districts and K-12 private schools are under increasingly competitive pressures. A simple model tells the story, divide the teacher's cost by the student-teacher ratio to get the cost per student. If, for example, the teacher's fully burdened cost is $100,000 a year

[21] Total college debt today exceeds total credit card debt!
[22] https://nces.ed.gov/programs/coe/indicator_cha.asp

[23] and the student-teacher ratio is 10 to 1[24] then the per pupil cost is $10,000 per year. As the competition increases, as the demands for additional services continue to increase, you can do the headmath. The business community, as a rule of thumb, multiplies base salary by 2.5 to calculate the cost of an employee or consultant and multiplies by 3 to 5 times to define the price they must charge for that person to contribute to the bottom line.

Education is a service business that has not learned to use technology to bend their cost curve, whose costs are rising and going to continue to rise faster than its customers can afford. Its business model is broken. No amount of tweaking, no efforts at normal cost-cutting or efficiency boosting of the current model will work in the long haul. If we are to make this business of education sustainable, we must change the model.

Educating the Future


The medieval classroom resembles our own.

It seems to be such a straightforward idea. **Education is about preparing our children for the future, for what they will do next. Yet, so much of our formal education today is preparing our children for the past.** They are prepared for tests in which they are told to take out a piece of paper and a pencil, turn off their phones and their computers, not talk to other students, and take an hour or so to write the answers to questions that demand they know the facts or skills required in the past. They do not show their proficiency in: working with others, using the Web to find what they need to know, practicing skills in analyzing complex real-world data, or using creativity to solve problems.

Their curricula, like their tests, remain segregated into subject silos: math, history, science, English, no longer separated in the world they will inherit, live, and work. The content and skills they are required to master they know perfectly well are no longer relevant because they constantly ask, “**Why do we have to learn this?**”

And in the main they continue to be taught as if they are empty vessels to be filled with information passed from teacher to student, grouped in classes of a modest size, sitting at

[23] This number does not seem out of line when NCES reports that the average base salary for public school teachers is $49,630. In business we would double the base salary to approximate the fully burdened cost.
[24] For example, the average K-12 student-teacher ratio in private schools is 12.2/1.

desks aligned in rows and columns with all facing the front to maintain eye contact with the teacher and not with each other in the fashion defined in the Middle Ages. In these classrooms the main speaker is the teacher and the main listener is the student. Students raise their hands in salute to be recognized to speak and are expected to show a common decorum and pay proper and constant attention.

The school day and school year remain agricultural era vestiges with little respect for the natural rhythms of sleep, seasons, or physical exercise. Those considered to be learning too slowly, falling behind, or not challenged enough are given an even longer, more intensive, and often more rigorous day. From the earliest grades, we divide classes up into the Robins, the Bluebirds, and the Cardinals to group students to learn at the longitudinal rate commensurate with their intelligence, energy, and concentration, though they are all supposed to learn the same things. This division continues into middle schools when groups are placed into common siloed classes where curriculum and instruction vary in rate and now in depth. By the time they enter high school our kids have been placed on the college/non-college scissors graph. And then onto college where they are further defined into those who will not make it, who will get their two-year degree, from those who will get a four-year degree, from those who will do “graduate” work and become “professionals.” Our educational system is still preparing our kids for a 19th century society and work labeled as unskilled, skilled, managerial, and professional. It is so rare that students make it across these imposed borders that we celebrate their success by labeling them “the first in their family to…”

We are so embedded in this education for the past that like fish in water we do not even see other possibilities. We cannot imagine an educational system that makes it possible for a large percentage of our population to attain the learning level we associate with a college bachelor's degree. We cannot imagine a system that enables us to educate the vast majority of our children from their early months through college at costs that do not bankrupt our nation or our children. We cannot imagine classrooms and schools that focus from the very beginning on communication, collaboration, creativity, and critical thinking even though these skills are considered high priority for the 21st century workforce. We cannot imagine enabling every child to reach their dream and to thrive in this new century even though we claim this to be our goal. **We cannot imagine this because we remain mentally wedded to a technology of learning that is nearly a thousand years old, a technology based on paper and the skills for using, storing, and transmitting data, knowledge, and practice in that medium.**

The Venerable Alarm Clock

The original personal alarm clock and “simple” technology

Technology has a consistent yet surprising cycle. It by making things more complex before it makes them simpler. Take the venerable alarm clock. Invented early in the 20th century, it sat by most bedsides to be wound each night and to ring each morning to start the day. It was easy to set with separate knobs in back to change the time of day and the alarm time, and separate dials on its face showed both. It was a simple device and easy for everyone to figure out. We would say today that it had a great UI.

A half century later, the first electric alarm clocks began to appear. They lost their hands and included a digital display as well as a radio. Now you no longer had to worry about winding the clock and instead of a ringing alarm you could wake slowly to music. But in return for this efficiency technology had made setting clock radios much more complex. These devices in our homes and hotel rooms required us to read complicated directions just to set our wake time. Their UI was so opaque that I found it easier to call the front desk for a wakeup call.

An early “digital” clock radio, technology made it more complex

Today, technology has made our lives much simpler. At home I need only tell Alexa to wake me up at a certain time, on the road I use my iPhone. New technology has given me more power and at the same time it has made aspects of my life much, much simpler. The story of the venerable alarm clock is repeated over and over again in technology. Over time as the technology gets more powerful and useful it can also cover up complexity and present a very simple, and to us humans, natural interface.

Teachers and administrators often worry about technology in education being complicated, costly, and fussy, viewing its broader future use through the lens of its current state. They worry students will have trouble with it or not tolerate the problems they have run into in the past. We are at that clock radio stage with much of our educational technology, finding it difficult to imagine what consistent usage and expected breakthroughs will bring. Computers and tablets are half the price of a year of school lunches and typically last for 4 years. They run all day on a single charge, are light enough for a kindergartener to carry in their backpack and are increasingly stable. Our Kindergarteners have been using tablets and laptops since they were babies, easily manage their UI, have learned to work around their idiosyncrasies, and know how to use them for communicating, calculating, and creating. We no longer can use the technology is a problem excuse!

Technology Levels the Playing Field

Sabrina (1954) showed the promise of technology for all today.

Sabrina is one, or perhaps I should say two, of my favorite movies. The first version was a Billy Wilder 1954 classic with Audrey Hepburn and Humphrey Bogart, and the second was a 1995 Sydney Pollack remake with Julia Ormond and Harrison Ford. Yes, I am a sucker for love stories, but I also love technology stories. These movies, particularly the first one, are fascinating for their portrayal of the lives of the rich and powerful, and Wilder and Pollack's visions of the interesting technology available to the rich. They help us to visualize the future with its democratization of that technology and, for us today, the future of education.

In the original version, we are introduced to the life of Bogart's character, Linus Larrabee, a rich and driven industrialist, who has all the privileges and expensive toys that wealth then could provide. He has a full-time chauffeur whose daughter Sabrina grows up above the garage next to the Larrabee mansion, a radio phone in his limo, a Dictaphone to make notes to bring to his secretary as he is driven into Manhattan to his office, a full-time secretary of course and a staff of assistants who get him tickets to shows, reservations for dinner and travel arrangements, and typists who beautify all his correspondence. He has a desk that lets him control the lighting in his office. And in its climax, he takes a luxury ship across the Atlantic to go to Paris to be with Sabrina.

Today, nearly all of us carry a cell phone. It lets us talk to our office, our family, or our friends. It can record and even transcribe our thoughts. The Larrabees could afford to hire an orchestra for their parties, we just make a playlist on our phone and link it to a portable speaker system. We don't need Linus's assistant pool to find the information we want, we just Google or Wikipedia it. We have a personal assistant like Alexa to order things for us or to make our reservations. And we no longer need a personal chauffeur to drive us around because Uber comes when we ask for it, and, the self-driving car, the truly personal car, will be here shortly, washed, waxed, and clean. Luxury cruises are now available to most of us, and we no longer need to spend a year in Paris learning French cooking, as Sabrina did, when we can get instant lessons on the Internet.

Technology has given almost all of us the capability associated with the rich and powerful just half a century ago. Technology has always done this. It has always enabled "average" people to have the advantages that had been affordable only by the wealthy. The wealthy and the movie industry executives could have their own home projection rooms and bring home films to project in their own screening rooms. Today, the bulk of us enjoy large high-quality flat screen TV's on which 500 cable or satellite TV channels, Blu-Ray disks, and a myriad of Internet channels give all of us access to almost any kind of movie we want, anytime in our personal screening rooms. Cheap paper and the printing press made books and libraries widely available. Indoor plumbing gave the population "free" servants to empty chamber pots. Automobiles enabled us to live in suburbs like the Larrabees and to have comfortable, personal transportation whenever we want it. Technology lets most us live the lives the privileged did a generation ago.

Can technology do the same for education? Can it provide every student the kind of education we dream about in the best private schools with wonderful teachers, the latest equipment, a stress on the arts, and the opportunities for our kids to be treated as the individuals that they are? I believe it can. Technology can increase equity and personalization in our schools. $200 good laptops are easy to find, and cell phones are ubiquitous. Free and reduced school lunches served to nearly 30 million students are reimbursed at about $3.00 each and snacks about $1. We spend nearly $600/year/student to feed their stomachs. Spending $200 to $300 per student every 4 years to feed their minds seems eminently reasonable. At the very least it should enable us to send our kids to public schools as good as the private schools Linus and Sabrina had for their children.

Effective, Efficient, Relevant

> *Light output and efficiency of blue and near-ultraviolet LEDs rose as the cost of reliable devices fell. This led to relatively high-power white-light LEDs for illumination, which are replacing incandescent and fluorescent lighting.*
>
> *Experimental white LEDs have been demonstrated to produce over 300 lumens per watt of electricity; some can last up to 100,000 hours. Compared to incandescent bulbs, this is not only a huge increase in electrical efficiency but – over time – a similar or lower cost per bulb.*[25]

LED 5 watt bulb

The first practical LEDs (Light Emitting Diodes) were invented in 1962. The first white light LEDs were developed a decade later. The first ones were tiny and produced very little light. Today, they have become the standard lighting devices in our homes and offices. They are cool, bright, cheap, and very flexible.

The first MR16 light bulbs were produced soon after the first LED's in 1965, and the ones we see everywhere with multifaceted reflectors were introduced in 1971. MR16s have been at the apex of lighting

25 https://en.wikipedia.org/wiki/Light-emitting_diode

since then. They are used in fancy fixtures, in ceiling lights, and in artwork spotlights. Today, we can still buy incandescent MR16's, and we can buy LED MR16's that are equally bright with the same color temperature. LED MR16's sell for about twice the price of the incandescent ones, but that price keeps dropping. Just a year or two ago they were 10 times the price.


MR16 50 watts

Their real efficiency is not in their price but in their electricity usage. A standard incandescent MR16 is rated at 50 watts. If it is on for 10 hours a day, it will use a kilowatt hour of electricity every two days. At a cost of 10 to 20 cents a kilowatt hour, it would cost $18 to run for a year at a low commercial rate. That is more than 10 times the cost of a bulb. The LED MR16 bulb uses 5, yes 5 watts of power. It uses a tenth of the power of the traditional bulb. If you run it 10 hours a day for a year you will have used about 20 kilowatt hours, or $2 worth of electricity. The LED bulb produces a trivial amount of heat, is safe, and its lifespan is 100,000 hours compared with 2,000-10,000 hours for an incandescent bulb.

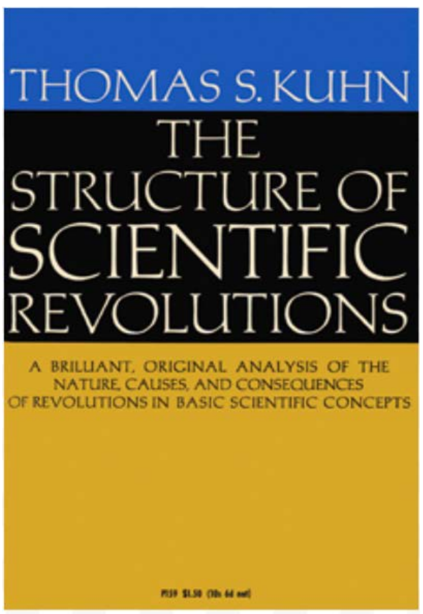
Light bulbs are just one example of the advantages technology always brings. It makes whatever we are doing more efficient, more effective, and more relevant. As the LED story reminds us, powerful new technologies can increase efficiency and effectiveness dramatically. They give us give us greater relevance, like more theatrical control over our lighting, enabling us to control color, timing, and Wi-Fi connectivity so that they can be turned on and off from a cell phone and even change their color temperature in the process, to “warm” a room at night.

This story has repeated itself in nearly every area of our lives since the earliest humans, and we have come to expect it. Sperm whales were driven close to extinction because their oil made candles that were nearly smokeless leading to the need for gas lighting and then the electric light. We have come to expect automobiles to use first less fuel, have fewer repairs, and more and more drive themselves. We expect our TV's to be thinner, larger, more beautiful, cheaper, and increasingly easy to get and find the programming we want. We expect our food to taste better, take less time to prepare, and to better fit our lifestyle. We expect to live longer, healthier lives with less wear and tear on our bodies. We have not yet come to expect technology to make education more effective, efficient, and relevant.

Though technology is broadly used in education today both in the office and in the classroom, its main purpose has been to automate repetitive tasks. Keep student records. Schedule students. Put lectures on YouTube so they don't have to be given repeatedly. Make a slideshow instead of writing the same things on the blackboard. Create video lessons on concepts so that any student can see them anytime they want. Keep a gradebook on a spreadsheet instead of copying results constantly. Give students homework exercises online for automatic feedback and grading. Put the syllabus into the learning management system instead of redrafting it year in and year out. Automatically test and assign to ensure that students are learning the lessons before they go on. Technology is used as a tutor, as a secretary, and as a copy machine, but it has not fundamentally changed the way students learn or what they learn.

The LED MR16 bulb is no doubt a significant advance in effectiveness, efficiency, and relevance but it is still in the shape of the old bulb so that it fits into existing fixtures. Unlike the original MR16 which changed the way houses could be lit, it does not fundamentally change the way we light our spaces. LED's offer us new lighting opportunities and creative ways to make the sunshine last longer. Perhaps it means windows, walls, or ceilings that light themselves. Perhaps it means glasses that enable us to see in the dark as well as we see in the light. This out-of-the-box thinking is just now starting to penetrate the lighting world. It is this lesson that technology offers educators. The opportunity to reimagine and reinvent both the curriculum, what students learn and instruction, the way they learn it, to bring the full power and imagination of technology to bear on the most important task there is.

The Structure of Revolutions



Paradigm shifts

"Is the Textbook Dead?"

It caught my eye, this headline and story posted in *EdWeek*[26]. Seems there was a panel at a conference that was supposed to debate what they obviously thought would be an attention grabbing, contentious, and controversial topic. Their conclusion: **NO!**

All I can say is:

"**You have got to be kidding!**"

Now, I know textbooks continue to play a central role in most of our schools across the grade levels. I know they have done so for centuries, I collect antique math textbooks. And I know both the textbook publishing community as well as the school community believe that paper textbooks will slowly morph into online "interactive" versions. Textbooks are so ubiquitous, so standardized, so traditional that most of us cannot imagine school without them. So, is it any wonder the panel came to its conclusion: the textbook, designed for print on paper (text is derived from the Latin for tissue) will always be with us? We may expect paper to morph into screens, but few imagine a fundamental change in form.

In 1962, Thomas Kuhn published a revolutionary work called *The Structure of Scientific Revolutions,* introducing the term "paradigm" into our lexicon. Kuhn argued that science changes, for the most part, as a slow continuous accretion of knowledge "normal science," but the history of science is also punctuated (to use Stephen Jay Gould's term) with "revolutionary

[26] https://marketbrief.edweek.org/marketplace-k-12/textbook-dead-far-early-last-rites-iste-panel-suggests/

science," times of dramatic massive fundamental change. We need only note the Copernican Revolution, the Newtonian Revolution, Maxwell's Field Theory, Einstein's Relativity, and Quantum Mechanics in the world of physics. These scientific revolutions introduce new paradigms, fundamentally new ways of thinking, that change the focus and direction of a scientific field. I would argue that technology, like science, grows in the same way. Most of the time it normally grows by small changes, incremental improvements, but every once-in-a-while its history is punctuated by revolution. The iPhone was not a mere smaller version of a corded or a better cord-free phone. It was a fundamentally new experience, a transformative experience in the ways we communicate. The integrated circuit changed the way we work. The Web revolutionized the way we learn.

The textbook, as we know it, an invention incrementally improved by the rotary printing press in the middle of the 19th century, provided a way for large numbers of students to "take a teacher home." Lacking interactivity, it did not replace a teacher, but for perhaps half of the student population it provided an effective supplement of class time with additional practice and information. It was not so much a tool for learning as a tool for practicing what you should have learned in class from the teacher.

What will the new paradigm for education be? What technology will replace the textbook and fundamentally alter student experience? That new paradigm is what this work is about. Digital technology with its amazing interactivity, its dynamic communication capacity, and its opportunities for collaboration, gives students powerful tools for learning. **What if...**

- We used digital age technology to enable students to learn on their own without the direct instruction of a live teacher?
- We designed a new paradigm for student's learning, tools not dependent on text or repetitive mind-numbing practice?
- We placed, into every student's hands, the interactive power of the Web to imagine all learning as a laboratory experience, a scientific experiment.

Before we can go there, we have to understand and empathize with the context for this new paradigm, to see and understand the problem created by technology that our new technology will have to solve.

Books vs. Courses

I have long loved Maxwell's Equations as the epitome of beauty in physics and as inspiration for my teaching. When I came across a paper by the great physicist Freeman Dyson called "Why is Maxwell's Theory so hard to understand?"[27] I could not resist reading it. His telling of the Maxwell Equation story has led me in new directions as I think not about physics but about education in the digital age. It has led me to ask the question, "What's the difference between a book and a course today?" And to ask, "What will they look like in the future?" But before we tackle those new questions, let's look at what Dyson has to say about Maxwell's great work.

> *In the year 1865, James Clerk Maxwell published his paper "A dynamical theory of the electromagnetic field" in the Philosophical Transactions of the Royal Society. He was then thirty-four years old. We, with the advantage of hindsight, can see clearly that Maxwell's paper was the most important event of the nineteenth century in the history of the physical sciences. If we include the biological sciences as well as the physical sciences, Maxwell's paper was second only to Darwin's "Origin of Species". But the importance of Maxwell's work was not obvious to his contemporaries. For more than twenty years, his theory of electromagnetism was largely ignored. Physicists found it hard to understand because the equations were complicated. Mathematicians found it hard to understand because Maxwell used physical language to explain it. It was regarded as an obscure speculation without much experimental evidence to support it. The physicist Michael Pupin in his autobiography "From Immigrant to Inventor" describes how he travelled from America to Europe in 1883 in search of somebody who understood Maxwell. He set out to learn the Maxwell theory like a knight in quest of the Holy Grail.*

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Maxwell's Equations

Maxwell's Equations in the elegant form on tee shirts and posters was not the way Maxwell wrote them down in 1865. He did not have the benefit and simplicity of vector calculus. And the idea of fields as environments was then brand new and hard to grasp. But of greater interest to me, beyond the significance and power of symbol systems which are well known, was Dyson's recognition that for many, perhaps most, revolutionary ideas, the value of putting them down in a paper or book form is not enough. They have to be taught; we have to learn them.

> *Pupin went first to Cambridge and enrolled as a student, hoping to learn the theory from Maxwell himself. He did not know that Maxwell had died four years earlier. After learning that Maxwell was dead, he stayed on in Cambridge and was assigned to a college tutor. But his tutor knew less about the Maxwell theory than he did, and was only interested in training him to solve mathematical tripos problems. He was amazed to discover, as he says, "how few were the physicists who had caught the meaning of the theory, even twenty years after it was stated by Maxwell in 1865". Finally he escaped from Cambridge*

[27] http://www.damtp.cam.ac.uk/user/tong/em/dyson.pdf

> *to Berlin and enrolled as a student with Hermann von Helmholtz. Helmholtz understood the theory and taught Pupin what he knew. Pupin returned to New York, became a professor at Columbia University, and taught the successive generations of students who subsequently spread the gospel of Maxwell all over America.*

I highly recommend you read the rest of Dyson's paper, but for now, as I suggested, I want to consider the question it has prompted for me. Fifty years ago, it was easy, very easy to distinguish a book from a course. A book was made of paper filled mainly with text, plus a few pictures, and bound between cardboard covers. A course was taught by a teacher talking to or with students collected in physical spaces. Today, they have all but merged. Books are read on plastic or glass screens, downloaded with links, sound, video, to interact with the world, and courses are available online, often without a teacher, and running from a few minutes to semester.

While we can no longer separate them physically, we may distinguish them conceptually. **We expect a book to be something we read through and a course to be something we work through, a book to be sequential and a course to let us roam, a book narrative, a course exploratory.** We will, I believe, see these two forms of learning continue to merge in our digital age, as education becomes more personal, as students take control over their learning, and as learning options and opportunities grow. You will, therefore, find in this "revolutionary" vision of the future of education both book and course, narrative tours like this chapter you have just finished, and interactive explorations to be found on our website www.makelearningreal.org.

You will, I hope, choose to participate in both aspects of this experiment, and treat this work as something to be read linearly, explored on the Web discretely, and created, collaborated, and communicated greedily.

2. The Aims of Education

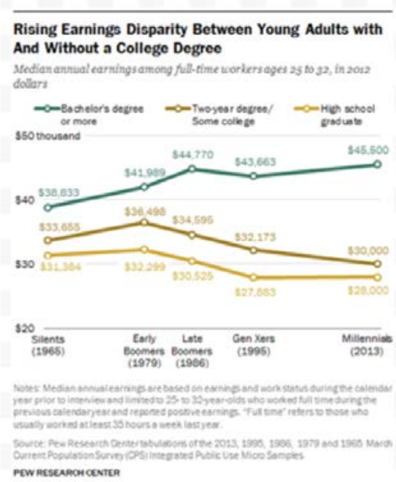
"Culture is activity of thought, and receptiveness to beauty. Scraps of information have nothing to do with it. We should aim at producing people who possess both culture and expert knowledge in some special direction. Their knowledge will give them ground to start from, and their culture will lead them as deep as philosophy and as high as art."

Alfred North Whitehead, *The Aims of Education* 1929

Our Kids will need a College Degree

Robert Putnam

I took to him almost instantly. Perhaps it was our common Midwest roots, perhaps our common age, perhaps our common way of thinking through visualization. What exactly made Robert Putnam's ideas stick so tightly to my brain I cannot say to this day, but since hearing him speak about his work last year and reading his book, *Our Kids*, I have been driven by his vision. Putnam would call himself a sociologist. He is a professor at the Kennedy School, Harvard's Kennedy School of Government, and a prolific author. In this latest work, studying the divergence of the middle class in America, Putnam documents in life stories of people, and in the statistical analyses for which he is well respected, the segregation of the middle class separating those families living the American dream and those that do not. He finds a singular variable to define this divergence, the college degree.



A Scissors Graph

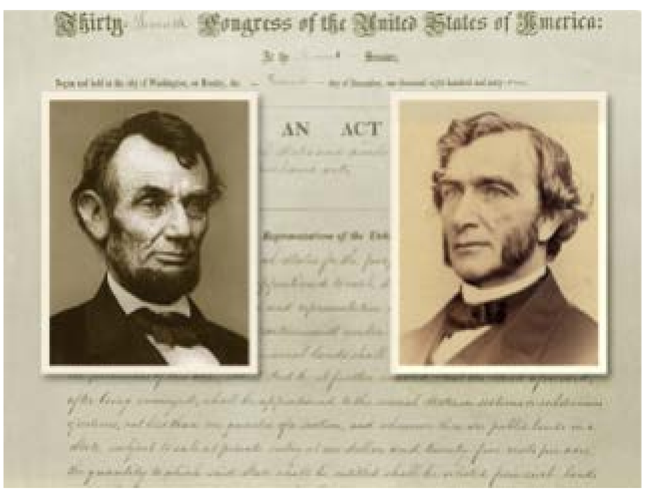
He showed the "scissors" graph with his fingers. Beginning in 1959 (the year we both graduated high school), the income of families that get college degrees and those whose highest degree is a high school diploma diverges. The world we both grew up in after WWII was fairly egalitarian, at least as far as education and jobs were concerned. My dentist father earned about the same living as our factory working neighbors. Our friends and families were mixed together. One of my uncles was a lawyer, another a doctor, a college professor, and a pop truck driver until he went to work in his uncles' small motor factory. I went to the public high school with the son of the developer of our community and the son of a state legislator. Putnam shows that earnings of managers/professionals in his manufacturing country were close to those of the good union jobs that proliferated after the War. The rates of divorce, drug abuse, and out of wedlock births were generally the same across the middle class, and measures we typically associate with a middle-class standard of life like housing, cars, meals, and entertainment were similar.

But as Putnam so vividly illustrates, 20 years later when our generation took the reins, the separation had begun. The college educated children had become the managers and the good paying union manufacturing jobs were starting to evaporate. Today, as the 3rd generation starts, their families and become breadwinners, the divergence has separated the "middle" class into two groups, those that are thriving and, if not living the American dream, still believing in it, and those that are not and no longer do. This past decade, after the 2008 economic collapse, has been particularly devastating. The middle class, whose average income dropped 10% from

$55,000 to $50,000, is now broken into two distinct groups, the upper group has seen their income grow and the lower group has taken all the decline and continues to do so. Putnam visualizes the income and education gap in other ways, in the percentages of divorces, babies born out of wedlock, drug addiction, bankruptcies, happiness, and in poll after poll that says those Americans no longer believe in the American dream, no longer expect their children to have a better life than they have, no longer see the future in a positive and productive way.

Putnam ties this divergence to a college degree. Of course, this is a singular and easily defined variable. Whether it is the cause or a proxy for other causes, we would not unequivocally say, but there is plenty of evidence that a college degree is a managerial degree associated with a problem-solving ability, abstract conceptual thinking capability, ability to learn and desire to continue to learn, and management discipline—the set of skills widely considered essential in today's business and STEM world. What we can say from his studies is this: **It is not college attendance that makes the difference, it is college completion that is key to a thriving middle class.** Unfortunately, we too often measure attendance and not completion.

What is a College Degree?



Celebrating the Land Grant College Act 1862
Abraham Lincoln and Justin Morrill

As an educator, Putnam's distinction makes complete sense. Our K-12 school system was designed in the second half of the 19th century to provide industry with a trained labor force for the repetitive work of manufacturing products effectively and efficiently, soon to become the iconic assembly line, orderly repetitive processes governed by clocks and standard procedures, and mastered skills.

Our K-12 schools have done this well. Today, most of our children graduate high school inculcated with the philosophy and trained in the methodology, ready to learn and perform such repetitive work tasks. Too bad these jobs will soon no longer exist, no longer provide adequate income, and will not be coming back no matter what some may wish and continue to hope for.

From the early 20th century, all kids were required to have a minimum of 10 years of schooling where they repeated paper-based skills again and again, practicing for their repetitive adult work. They sat in fixed rows, raised their hands to speak or to perform other tasks. They moved from class to class by the clock and the bell and were given short recess opportunities to stretch their legs. They were led by a teacher, the classroom manager, who provided the work, measured the output, and kept order. **Our schools today continue to be relics of that**

Industrial Age education. Kids still learn the reading, writing, and arithmetic to enable them to participate in repetitive work, learn additional skills, and form a disciplined well-ordered workforce. They are not learning to be managers, though an increasing number are trying to prepare to be part of the managerial class.

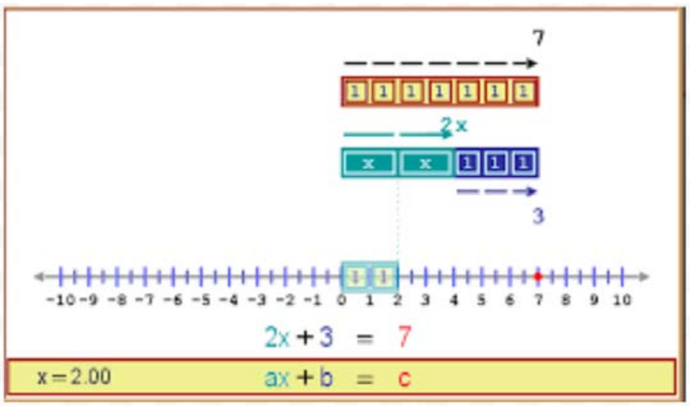
Our college system, beginning about the same time as represented by the Morrill Land Grant College Act, was designed to produce the managers, people who would run those factories, the professionals who would run their own businesses, or the engineers who would solve new problems. A college degree was designed to mean that a graduate could solve problems and manage a project or a business. Land Grant colleges, funded by gifts of federal land to the states, and a myriad of private nonprofit colleges opened their doors after the Civil War to educate those managers, to ensure they could effectively communicate, collaborate, solve problems, and bring creative skills to their businesses. They sought to provide not only training in the arts of business, but in the liberal arts as well, so the managers they graduated could instill art and culture into vibrant and engaging communities. Over the next century, the middle-class managers grew substantially along with the unionized manufacturing labor force.

Since the end of the Second World War, less than 75 years ago; though college graduation rates have grown to 40% of our student population, virtually none of that growth has occurred over the past decade. More than a third of our students entering college are burdened with required remedial classes to “prepare them” for college. And our business community complains that those who make it through, who finally get a college degree, are not prepared for successful managerial work. They have not learned what they need to learn. So, it is not enough to try to grow the number of students that our colleges graduate with bachelor degrees unless we also improve the effectiveness of their learning. Business leaders do not find today’s managers to be as creative as they would wish, as good at solving problems as they would like, or as able to communicate and collaborate as they need to be. It is clearly not a matter of making more managers in the 19th century style, it is enabling more, *many* more people, to gain the managerial skills, to be prepared to live and work in the digital age.

I rarely heard the term entrepreneur used before the 1980’s. I never, as a child, imagined myself an entrepreneur. Yet, I am a 3 times entrepreneur, and I have been an investor in and mentor of other entrepreneurs. The digital age is the age of entrepreneurs. And though we usually think of them as associated with high tech, that need not be the case. Open a pizza shop, become an electrician, be an organic farmer, or invest in a lawn tractor and start mowing lawns for a living, and you are an entrepreneur. You are managing a business or a product. You are planning, communicating, dealing with finances, and with legal issues. Most managers, or people with managerial tools in the digital age, must act like entrepreneurs in the fullest sense of that word, creatively solving problems, making decisions, organizing, communicating, collaborating, and continuing to learn their craft and trade. The digital age is the entrepreneur’s age, and our job as educators is to prepare our students to thrive as managers and entrepreneurs.

So, I ask you to try to visualize their future. It is not easy. Things are changing so rapidly. Yet, I do know for certain, when I do this exercise myself, that we are obligated to set up two goals for their education. The first is, at the very least, **to prepare our kids for the present and not the past**. The second is **to prepare them to be lifelong, continuous, flexible, creative learners**, to be able to best handle the future whatever it may be. Both goals require, at a minimum, the kind of education that prepares them to manage, the kind of skills college bachelor degrees were designed to develop, on a foundational level they are the 4Cs, (Critical Thinking, Creativity, Communication, Collaboration), the skills the business community tells us they are looking for not only in their managerial employees, and their entrepreneurs, but in *all* their workers.

The Developmental Math Problem



EnableMath Concept screen for solving linear equations

When I picture what so many of our kids face as they try to get a college education, I see the dingy small office of the math department chair at Baton Rouge Community College I visited a decade ago. I was there to show him our EnableMath[28] solution to his developmental math problem. Developmental math is the euphemism for the non-credit remedial courses taught in colleges to theoretically prepare students for their required college math courses. The chairman was trying, his small office filled with paper that appeared to me to be a growing pile of unresolved problems, while his day was filled with administrative and personal detail like finding a key for an adjunct to open her classroom door, joining some faculty for a birthday party, or dealing with a student who had a scheduling conflict. I sat patiently as he dealt with issue after issue until, finally, he had a break and turned to me.

I asked him, as I and my colleagues asked more than 100 other college and community college presidents 10 years ago, “What are the success rates in your developmental math courses?” The math chair looked up at me, shook his head, and said that he did not know, but guessed it was significantly under 20%. He turned to a faculty member he had invited to join us, who suggested that the number was more in the 36% range. I was stunned. I had been hearing numbers and seen national studies with 50% as the average success rate in each and every developmental math course a student had to take. The California Community College System which had studied the problem, estimated that students thrown into the developmental math sequence had an overall 18% chance of getting through it successfully. Percentages are often hard to feel, so to put these into context, consider that in most regular college courses success

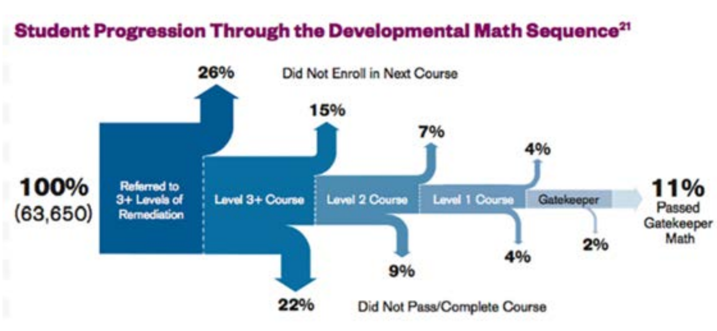
[28] EnableMath, a product of Enablearning, Inc. was a fully online solution to the developmental math crisis. It included dynamic visual concepts, interactive step-through examples, and individualized problem sets with adaptive mastery practice.

rates are at the 85% level. If a teacher "grades on a curve" to be totally fair, she will fail 15% of her students!

The numbers, as obscene as they are, do not tell the real, the human, story. For the year before, I had been in a developmental classroom in Massachusetts watching a very good teacher use our EnableMath program. I got a chance, after her class, to talk with one of the students. I remember a beautiful Latina in her mid-30's who told me that she hoped this class would be the charm. She wanted to be a social worker but first needed to complete her AA degree. She was taking this Algebra 1 developmental math course for the 3rd time, and if she failed, she was going to give up. She was praying that this time she would make it, that this gate would finally open for her. I can still feel her dream for a better life, her passion, her desire to this day. Her dream became my dream.

It is the dream of the vast majority of our kids that Stanford University Bridge Project reported in "Betraying the College Dream" in 2003 after surveying thousands of students. **They found that 88% of 8th graders aspire to and expect to participate in higher education.** We have sold them on the value of a college education, but most can't and won't get it. Developmental math and its partner in ELA[29] remain the primary cause of student failure in community and 4-year colleges, I believe this is the biggest and saddest single problem in all of education, killing the dreams of more than a million of our children each year.

The Odds are 1 in 5



The STEM pipeline!

How do those odds sound to you? If they were your chance of winning a million dollars, they look great. If they are the odds that you will get some disease, they are dismal. Well, those are the odds that a student will successfully learn the math they are supposed to learn in K-12. One out of every 5 students who take 12 years of math in our elementary and high schools will have the math understanding and confidence they need to use and apply math in their working lives, 1 in 5.

How did I get that number? It is an easy calculation, and it is based for the most part on actual "scientific" data. First, start with the percentage of students who graduate high school and go on to college. That number today is in the neighborhood of 66%, 2/3rds of our kids *try* to get a

[29] English Language Arts

college degree, they know how important it is. Now I know one thing about the 1/3rd that did not try to go to college, they did not think they were good enough in math to make it through. If you are good in math, if you get math, if you are confident that you learned and can learn math, then you were in all likelihood in a good track in high school and somehow or other you are going to try college, and at least get a 2-year associate degree.

Of those 2/3rds who try to go to college, over 1/3 of those, actually 36%, fail to qualify to be ready to take a college math course which is generally a college requirement. We are not talking about calculus here. We are talking about college algebra. Entering students qualify to take a for credit college math course by scoring well on the ACT/SAT tests, or by passing a college placement exam like the Accuplacer from the College Board. So, 1/3rd of 2/3rds is, if you calculate it, about 2 in 5 students. Around 40% of all our kids qualify to take a *college* math course leading to a college degree. Tragically, less than 20%, 1 in 5 of those that must take developmental math courses to ready them for college math ever succeed in even taking, let alone passing a college math course. And 40% is the percentage of our children who today get college degrees. This number has barely budged for the past two decades.

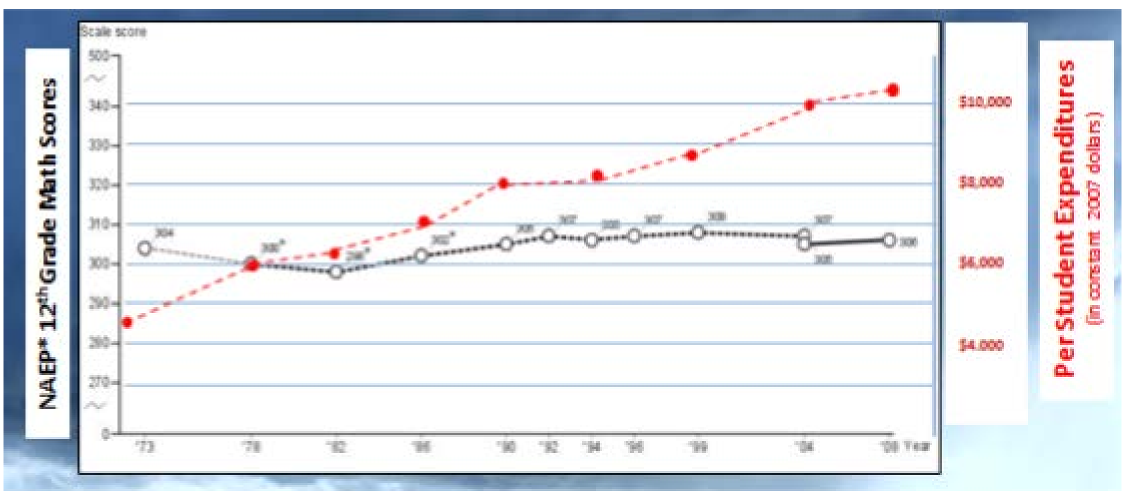
Now one more step. Of this 40% who pass at least one college math course, my “experience” and here I admit I am not being “scientific” but base my calculation on a great deal of personal experience, that about half, and I think I am being very generous, get the math they were taught. As a former math teacher, people constantly have the need to unburden themselves, to use me as an excuse to talk through their feelings about math. By my reckoning half of those people tell me they may have been taught math, even calculus, but they don’t get math and really don’t know how to use it. So that’s where I get to 20%, **the odds that 1 in 5 students who have sat in math classrooms for an hour a day for 13 years and more at home (2,000 hours, likely double that of their lives, a person-work-year, have even learned the rudiments of the math we want them to know.**

We can blame this tragedy, this waste, on teachers who themselves are in the 4 out of 5. We can blame it on the way we expect math to be taught. We can blame it on kids not caring, on calculators, on bad schools, on TV or video games. We can blame it on the irrelevance of the math problems they are supposed to learn or on our lax culture. But before we assess blame, before we charge once again, headlong down a path to fix a problem that has not been fixed, we must look at the math we want our kids to learn and ask in each case whether it is necessary for 21st century success, because 1/5th is a very small number and it has not gotten better despite a substantial history of concern and effort. It is clearly not time for another incremental improvement, a new trial of some tweaks. It is time for us to reinvent math education, for math is more important for the digital age than it has ever been, and more important for all of our kids.

Can We Expect it to Get Better?

I wrote this a decade ago. Little has changed.

> *We worry about literacy in our nation and yet most adults can read. As important as reading our native language is, and I do not discount the need to improve reading at all, the true failure of our educational system is patently obvious in math. Despite the familiar refrains of the many critics of our math educational system, we all have to understand that as a nation we have tried to fix this problem. We have sought to bring in many different new curricula. We have tried to focus on procedural competency with workbooks and worksheets. We have tried to make math much more real through manipulatives and authentic problem solving. We have tried to educate teachers better and to cookbook creative lessons. We have tried to spend more money against the problem. And we have been making a greater effort to use technologies in our classrooms. But we have failed and failed miserably. The NAEP scores in 12th grade mathematics, the gold standard for judging math performance in this country has not changed in 40 years. Look at this graph, it is flat, it has a slope of zero.[30] And as I like to tell my friends in math education, zero*



NAEP 12th grade math scores are flat from 1973-2010 while per pupil expenditures more than doubled

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[30] Some would argue that this is not an accurate representation of the problem, that the student population has changed, that we no longer give the 12th grade NAEP tests because students don't care about them and do poorly on them as a result and therefore that the latest scores are approximations. I would not dispute these criticisms. They would be valid points against my argument if you were trying to argue that differences of a few points or a few percent were significant, that we were really making progress. It does not. Changes of a few percent over this long period means, for all intents and purposes, that we are stagnant, even regressed. I find the same tragic argument used by state officials about progress made on the statewide tests. A percent here, a percent there means so little, when we require 100% improvements. As Galileo was so fond of saying about balls dropped from the Leaning Tower of Pisa, "We cannot hide the two cubits of Aristotle behind two fingerbreadths." We cannot hide the massive failure of our education system to enable every student to learn mathematics behind a percentage point here and a percentage point there.

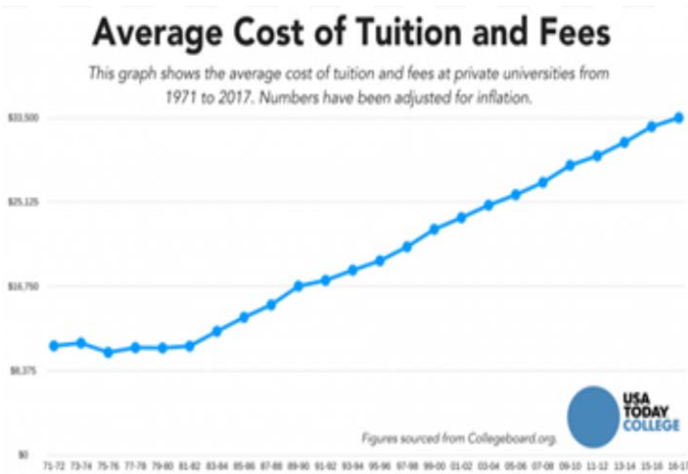
can be a very, very big number. At the same time, our overall expenditures in constant dollars has nearly tripled!

The National Assessment of Educational Progress (NAEP) is considered the gold standard of tests. If we add the latest results to this graph, we will continue to see a slope of 0, a lack of any progress, no significant improvement. We can say that we must try harder. We can say we need to spend more money and get better teachers. We can say we must do more “scientific” research to understand the problem. But during this same 40-year period the amount of money we spend per student (the red line) in real dollars has nearly tripled. Our failure to improve student math scores is not due to a lack of will or a lack of trying. I am ready to take off my football helmet, it doesn’t really work anyway, and stop banging my head against a concrete wall. I have studied math education for 50 years, I have tried most of the new ideas myself with kids and with teachers. And I cannot say why some kids get math and so many don’t.

Of course, we can’t give up. Mathematics is, if anything, more important in the digital age than it has ever been; mathematics to solve complex real-world problems is essential for every one of our kids. But before we go out to try again, we have to look at what it is we are asking our students to learn. **Is it the math needed in the 19th century or will it be the math needed in the 21st century?** As we will soon find out these are two very different things.

Do our kids need to know long division, or how to divide fractions? Do they need to be able to factor equations or use the quadratic formula? Do they really need to be fluent calculators on paper at all? Picture the math you learned and the math you use daily, is there any correlation? We will soon find that most of the math we force our kids to learn is irrelevant and obsolete. It neither builds understanding, nor does it support problem solving. If we start from scratch to design a math curriculum for the 21st century, a curriculum that uses Web and spreadsheet technology as primary tools, I believe we stand a very good chance that 4 in 5 of our kids will actually learn it and be able to use it to solve their digital age problems. But before we go there to make math education and all of education for that matter more relevant, we need to make it more efficient and deal with the great problem of the cost of education.

Higher Education is Unaffordable



Average cost of tuition and fees at private universities from 1971-2017

Moderately selective colleges, private and public, are facing an existential crisis. They are, or soon will become, *unaffordable*. Just at the time when a college degree has become an imperative to be a member of a rising middle class, just at the time when the technology revolution is reinventing work and workplaces, just at the time when the need for a college degree and for the kind of thought processes and skills long associated with college degrees have become the credentials for good paying jobs with managerial or creative opportunities; we are driving these degrees out of reach of our kids. Tuition discounting, now averaging nearly 50% and growing, approaching 70% in some colleges is unsustainable and unaffordable.[31]

Let me use my personal experience to put college costs into perspective. I entered the University of Chicago in 1959. I remember well my tuition and room and board, each cost about $300 a quarter for each of 3 quarters of the traditional school year or $900 a year for tuition and as I recall $960 a year for room and board, $320 a quarter. Though, I did not then pay serious attention to money, I was able to pay the Bursar on a quarterly basis with a small scholarship from the University, a student loan, a science prize scholarship and some help from my parents. My father was a dentist and earned about $10,000 a year, a middle-class income.

Today, room and board at UChicago is just over $15,000 a year. Factor in inflation (about **10x)**, that number represents reasonable growth rate of 1½ times inflation, and I am very certain both the food and the accommodations are substantially better today than our miserable cafeteria, tiny spare rooms, and communal bathrooms. Tuition at UChicago today, however, is another story. It is just over $50,000 today. Tack on a student life fee of $1500 and books and personal expenses of $4000, “tuition” exceeds $55,000 a school year. A **60x** increase. Let me write it out in words – tuition and fees at the University of Chicago have increased more than sixty times over the past 60 years while inflation by most any measure has increased the cost of even college living by the 10 to 15 times. The cost of college has not been driven up by the cost of

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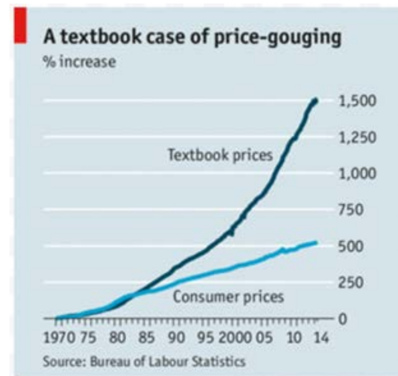
[31] Don’t tuition rates typically increase because of inflation? Yes, and this year’s increase — as has happened in the past — is higher than the rate of inflation. In the past 12 months, the rate of inflation stood at 2.2%, according to the U.S. Department of Labor. Colleges, however, appear to be increasing their tuition rates by nearly *double* the inflation rate — a trend that has been consistent for the past decade. USA Today *By Kellie Bancalari 2:28 pm EDT June 9,17 20*

fancier dorms and better food, it is being driven up by tuition. If we consider room and board increases as a measure of “real” inflation, then tuition and fees increased **4X** that rate.

The grounds and the renovated classrooms are much more beautiful than the sad campus weed filled trampled grass and dark dingy classrooms I thought normal. But I cannot imagine they have improved the learning. Nor, can I imagine that the quality of the faculty has profoundly increased. I loved my U of C education and treasure it to this day. Though I and my classmates may consider changes in the liberal arts (The College) requirements even a step backwards, I doubt the quality of education has changed substantially either for better or worse. Certainly, there are new costs, technology-driven costs like high speed Internet in every classroom and video projectors in most, but there are also technology-driven savings in duplication, maintenance, and administration. And there are new costs in student services that have little to do with student learning, but I doubt that those would even come close to doubling the general inflation rate, much less quadrupling it.



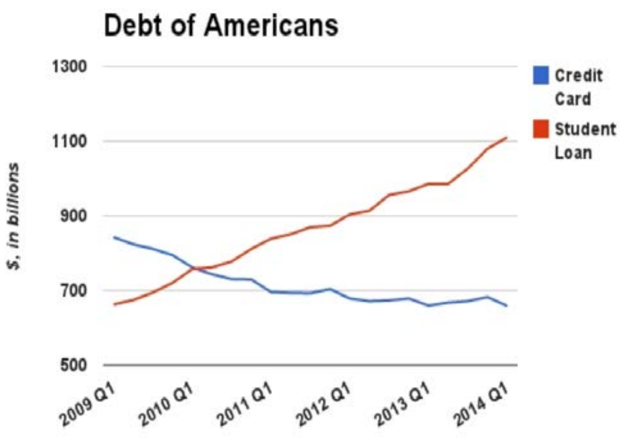
$250 textbooks are not unusual

Part of this ridiculous rate of increase in college costs is driven by a fiction. As tuition has increased so too has the discount rate. Colleges are giving “scholarships” to students to make tuition and fees affordable. They publish a stated tuition when in reality the amount of money they expect to collect and spend per student is about half of that number. This discounting has been going on for a long time. I do not know what it was in 1959, but today colleges are like those flooring commercials that cover cable channels announcing 50% discounts on new carpet installed in your home. Time to rush out and get these amazing offers. Even factoring in the discounting, college tuition and fees have increased at nearly **3x** the rate of dorm inflation. This is true across colleges of all kinds, public, private, nonprofit, and even for-profit. And this increase in costs has not substantially improved the product, our kids do not learn more today, do not learn faster, come out of college able to think better, or perform our jobs better. Our kids just come out of college in debt.

While technology has already profoundly changed our world, and promises even more accelerated change, our higher education system clings to its medieval model. Though, it has selectively applied technology to both management and instruction, it has only succeeded in automating long-held processes. It has made little progress in inventing or applying new ones. Virtual, or so-called online courses, generally still have the same teacher student ratios as classroom courses.[32] **So, we remain mired in a system of higher education with static productivity, exponential growth in costs, and increasingly fierce competition for the 40% of students who likely will succeed and the 3% of students who can pay full freight.**

[32] Colleges like SNHU, Arizona State, and Georgia Tech are experimenting with new models, but it is not at all clear that these will work for students who traditionally have not had much college success in standard college classrooms.

Rate of Change



This picture is worth a 1,000 words

It is all too easy to claim there is no cliff, only an increasingly difficult environment. It is all too easy to claim that we really don't need "everyone" to be a college graduate and that we should be thinking much more about trade, skill-building, and other job training for those unsuited to follow the college route. It is all too easy to make the claim that this great lumbering giant we call higher education, with a thousand years old traditional design, cannot and will not change.

The history of change in both business and technology provides very different evidence. **Businesses that do not meet the challenge of a new technology do not survive.** And they collapse with surprising swiftness. Institutions that think they are immune to change, when faced with the demands of a new technology, either change or disappear. And those of us who continue to think that linear change can enable them to predict and prepare for the inevitable are always surprised at the rate of change of the "tsunami" when it does occur.

For example, if a college raises tuition by just 3% a year, many of us naturally think that such growth is a linear function, slowly and surely increasing at a constant rate. But it is not, it is a compounding function, an exponential function that in 10 years will not add 30% to the cost of college but more like 50% to the cost of college, and in 20 years double the cost of college. So, if you are a good middle-class parent who creates an education fund for your kids, and you put in enough each year from the time they are born to pay for the current cost of college, think again, for that cost will double if we continue at the present rate! Even if you are rich enough to fill the fund at birth with the $400,000 list price of an undergraduate education, and put that money in a savings account, it will earn about 3% interest and though doubling in value, it will still only barely cover the retail cost of that BA degree. Only 3% of our families can afford the list price of a college education.[33] Three percent is not a sustainable future.

Student debt is already causing our colleges to compete for full-pay students, often from abroad, and as students' rebel against assuming increasingly catastrophic debt, our colleges will be competing for a decreasing number of customers who can afford our product in any way. We are heading for a cliff, and we know it. What do we do about it?

[33] Tom Williams, who had been president of the higher ed consulting firm Noel Levitz in personal conversation.

The Achievement Gap[34]


It hasn't really changed a lot

Higher education is not alone. K-12 costs have risen more than 3 times the rate of inflation. In constant dollars, the average per pupil cost of K-12 education has risen from a little over $3,000 per student 60 years ago to about $11,000 per student today. This increase is in part due to increasing teacher salaries, bigger administrations required for an increasingly complicated demanding world, and most of all a *decrease* in student/teacher ratio from 26 to 1 in 1960 to 15 to 1, nearly halving average class size. We have believed for the past half century that smaller classes with more individualized instruction time per student leads to better outcomes. We saw it as the way to optimize the standard model of education.

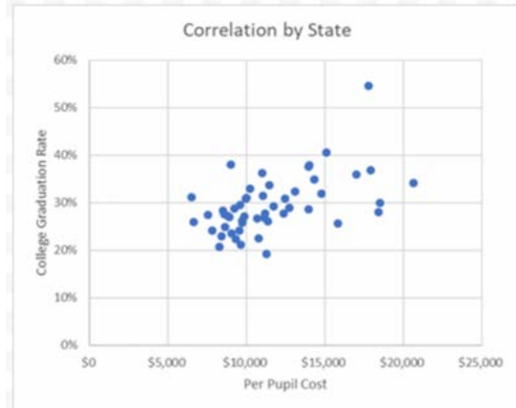
We began this trend when we unbolted the desks, freeing them from the floor, first in the rapidly growing suburban districts and later in the city schools. Mobile desks doubled the area each one required, for they no longer automatically fit into neat and tidy rows and columns. Schools had a choice, either make classrooms larger to accommodate mobile desks or cut the number of students in half. They chose the latter following the private school model of improving education through more individual teacher attention. At the same time, teachers who had been paid off in decent pensions rather than decent salaries, began to clamor for the good middle-class life of their students' parents who more and more, saw education as the key to ensuring their children were upwardly mobile, pushed for better teachers which meant higher pay.

The right of access to the better American life, extended to black as well as white Americans and to those who for one reason or another had learning issues. These demands became embedded in our educational belief system and led to continuously inflating educational costs by seeking to usher in an era of equitable schools and of special education which decreased student/teacher ratios and increased school budgets. These increasing teacher costs are only a part of the problem. Other school costs have risen much faster than overall inflation as well. Textbooks, technology, and testing have been dramatic drivers of school costs since the 1960's, while adding surprisingly little additional value to student learning or student success. And compliance with new norms, mandates, and requirements from providing transportation to school lunches, from sports activities to the arts, from equipping modern science labs to meeting safety standards are all-natural drivers of inflated school costs.

At the same time, there have been very few ways for schools to become significantly more efficient and economical. Technology has been brought into the front office, but I rarely see

[34] Sometimes called "The Opportunity Gap." It is no doubt exacerbated by high per pupil expenditures.

signs that it has saved money or increased efficiency. Schools have been much slower than business in reducing the auxiliary labor force, for example, that we have long called secretarial. They do not or cannot plan investment in efficiency to produce long term savings. School budgets continue to operate year-to-year and school administrations continue to be judged on each year's individual performance and in response to the pressures from both community groups as well as teacher unions.[35] Today, our K-12 schools continue down the same path saving a little money here and there while the great wave of inflating costs sweeps ever higher, just as they do in colleges..



Correlation between state expenditures for education and college graduation rates

Do these high and increasing per student costs exacerbate the achievement gap so detrimental to our minority population? I think a good argument can be made that they do. With paper technology, schools are dependent on instruction and thus on the quality of teachers, class size, and support services. The per student budget is likely the key variable in student achievement. The correlation by state between per pupil K-12 expenditure and graduation rates is very high[36].

In PK-12, high and rising per student costs affect our inner cities and rural areas disproportionally. Affluent areas of cities and suburbs, which consider schools as critical infrastructure, may well be able to afford the increasing costs, or add additional costs as a smaller percentage of their overall budgets, but the communities already under financial stress must find it more and more difficult to manage high and growing costs.

The cost of education is a determining factor in college success rates. If we compare per pupil cost with college graduation rates by state, we find a correlation greater than 0.5. This means that over a quarter of all the variance in college success rates across the U.S. is due to the amount of money the schools in each state spend on education. If we rank the states in their educational spending per student over the past 50 years, we see very little change. I can only conclude that among the variety of efforts to improve PK-12 education leading to college success, wealth, or at least the willingness by a community to spend its wealth, has played the central role in determining college graduation rates. The top 10 states in college graduation rates spend nearly twice as much as the bottom 10 states do per student and have nearly double the college graduation rates. Does this have to be? As we will see in other areas of our lives, new technologies profoundly change the relationship between rich and poor, between haves and have-nots.

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[35] I wonder what would happen if schools were forced to develop 5-year budgets?
[36] Pearson r is 0.53 by author, from state and federal data.

Can We Even Meet our Core Expectations?

Gutman Library

The basement conference area at Gutman Library of the Harvard Graduate School of Education was filled. Over 200 people showed up to hear a talk on the then new ELA Common Core Standards by a former Ed School graduate and now professor. As an interloper from the math world, I sat near the back and found myself mesmerized by a terrific talk. Her slideshow on the other hand left a lot to be desired with screens full of bulleted text and tables instead of graphs. Sitting in back, the PowerPoints[37] were a blur, but I doubt they were substantially better in the front row.

I went up to her after the talk and waited patiently to tell her how much I enjoyed it and to offer PowerPoint expertise, a skill I had long nurtured, to help her improve the slides. But by the time I got a chance to talk to her my courage had evaporated, and I decided to ask her my nagging original question instead. **"What percentage of students today do you think could pass tests on these standards?"**

I had been surprised at the demands I thought they were making on students. "About 10% she blurted out!" I was shocked. These standards were aspirational, they were not real. "What do you think?" she asked me in return. I told that I was a math guy and that I thought the Math Standards were equally ambitious and likely less than 20% of our kids would pass them if the tests were written to really assess the learning objectives. She grabbed at the higher figure and suggested that might well apply to ELA as well.

The standards movement had been designed as the baseline for all our kids. They were, after all, to be "Standards" for all. They had become "Aspirations" for our top academic students. To make matters even worse, the math Standards[38] have 2 parts, there are math content standards, the curriculum topics we know and love, and 8 math procedural standards to define the thought and problem-solving processes we hope our students learn. In ELA a similar distinction requires students not only to be subject matter proficient, but to have cognitive skills of the highest order.[39]

It is well to be ambitious and to have high hopes and dreams for our students. It is quite another to promulgate a fictional world and to use it to evaluate our kids and to prepare them for their future. I don't fault the writers of these standards. They rightly see the world that we live in as much more complex than it used to be. And they are trying to make our educational system

[37] Microsoft Corp
[38] Math Common Core State Standards Link
[39] I refer here to Bloom's Taxonomy of the Cognitive Domain

reflect that world. But without taking the bigger picture into account, they cannot keep adding to the burden without taking anything away. Before we can enrich our standards; before we can ask what future schools should look like to educate our kids; before we define our ultimate goal–like learning how to think, work together, be lifelong learners; before we can make our goals the 21st century skills 4Cs (critical thinking, creativity, communication, and cooperation); we should understand what in our current educational practice and expectations we need to keep and what we need to discard. Adding digital age skills and knowledge to our already overburdened schools does not and cannot work.

Over the past 50 years we have moved the percentage of our students who we expect to go to college from 25% to substantially over 50%. We expect almost every student to become a fluent reader and writer. We expect almost every student to learn algebra by the end of 8th grade and our better students to master AP Calculus by the end of their 3rd year in high school to use those scores for college admission. And we keep adding to those expectations, learn much more science, some technology including coding, a basic understanding of engineering, and of course mastering algebra 2. We expect them to have a deep understanding of history, civics, psychology, and economics in a world that has nearly quadrupled the number of countries in the United Nations and now includes all sexes, religions, and areas of the globe. We want them to have experience in the arts to make them "well-rounded". We, of course, want them to understand interdisciplinary global problems like climate change, and to participate in their community's and school's activities.

We expect all these things even though the school day remains at about 6 hours, the school year remains at 180 days, the content continues to come from textbooks, most school work is delivered and passed in on paper technology, and the learning process has changed minutely, if at all, over the past 50 years. As job opportunities for women and men of color have broadened dramatically, teaching in K-12 schools has become a less competitive profession populated with people of generally lower academic achievement.[40] And while we have continued to move the deck chairs around on the Titanic by fiddling with the curriculum over the past 50 years with the Common Core being the latest of these so-called reform movements, a teacher time-machined from 1917 would, except for the grade level and perhaps for the examples used, find today's classroom very comfortable and today's curriculum fully recognizable. What would you throw out? For without substantially changing the quality of our teachers, the pedagogy, the curriculum except to make textbooks bigger and more colorful, the time students spend in classrooms, or the genetic makeup of our kids, **why would we expect them to be able to learn so much more, so much faster, at a much higher level, and with greater retention**. In the next section we will consider what we can and must throw out as a first step to enabling all of our students to learn and retain much more.

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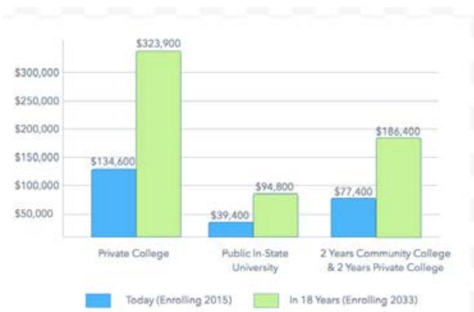
[40] One of my favorite numbers is 484. It was the mean SAT score a decade ago for students who completed their survey and said they planned to go into teaching. 484 was about half a standard deviation below the mean of the entire test. It was their mean score in both math and reading. I would be surprised if it has changed substantially today.

Our Kids

In conversation after conversation I hear the same four laments. We have become convinced that this huge ship will not be turned, that the expectations of politicians, teachers, and parents make educational change nearly impossible. But their inertia is nothing compared to that of our kids. Despite their acceptance of technology. Despite their concerns about the future. Despite their distrust of the existing state of affairs. They will not move, they will not demand, they will not require the changes we need in education. The laments come down to these.

Can't afford it!



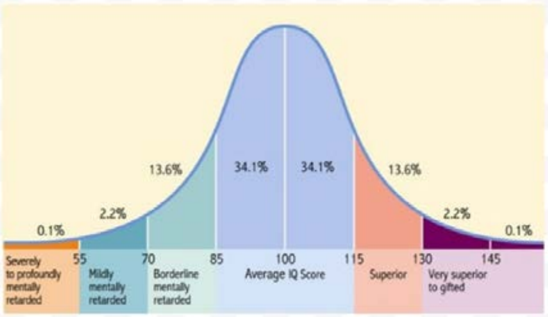
College is unaffordable and forecast to get worse.

Since the great recession in 2008, we have heard from students, parents, and citizens, "Is it worth it?" Is the college degree worth the cost? Is it worth it to take on so much debt? Will my college degree job pay me back for the time and the expense of going to college? As the cost of higher education continues to grow exponentially,[41] as students question its relevance, and as more and more people question its efficacy, it is not at all surprising that our kids should be asking this question. They are questioning their ability to afford it.

The economics are not trivial. The average student debt in 2016 in round numbers was $40,000. At say an interest rate of 7%, that would cost a student nearly $3,000 a year just interest alone, plus $2,000 a year to pay off the loan in 20 years. That $5,000 is the difference between the income of the average middle-class family with and without a college degree (20 years at $5,000/year the difference between an average income of $55,000 and $50,000 per year). Halving the cost of education changes this equation profoundly making a college degree affordable.

[41] It is currently growing at about 3% a year after going up nearly 6% a year in the early part of the century. This is exponential growth and as we will see impossible to sustain.

Can’t Hack It!



The “Normal” curve captures our IQ prejudices

The view that many of our kids are just not capable of doing true college work is widespread. I hear from educators and non-educators alike the “we need electricians” refrain. I hear that not everyone can or should go to college. We have come to believe as a society that it takes a certain level of intelligence to get a bachelor’s degree. Codified as around 110 on IQ tests, it suggests that everyone below that should not be made to strive for a college degree because they are just not smart enough. We have come to broadly accept this bias, believing that even an average IQ (100) is just not good enough to make it through the rigors of college reading, writing, calculating, and understanding. First the IQ tests and then the SAT’s were designed to measure this ethereal capacity to handle a college education.

But, like every other human skill, this capability is a function of technology. Before amplification a politician with a soft voice would not be a good speechifier; a horseback rider would likely not make a good auto racer; or a COBOL programmer a good Python coder. The metrics we use for intelligence today measure those capabilities demanded by higher education today, the abilities to read, write, and calculate. If we change the primary technology of education as we have the primary technology of business, we also need to ask whether those will remain the necessary primary capabilities, for they are fundamentally different from the 21st century skills business is looking for where communication using interactive images is likely to be a much more valuable skill than the ability to write long essays.

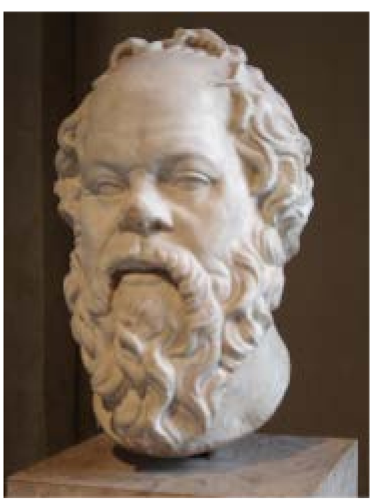
Don’t Need It!

Perhaps the most devastating lament is this one. In a recent episode of “This Old House” stalwart of PBS and a must have for many of us, Norm Abram ended the show, veering from typical show format talking with Mike Rowe pitching for young people to go into the trades. Not everyone needs to get a bachelor’s degree they argued, college is not for everyone, you earn a good living doing useful work in a trade. I hear this argument often. Not everyone, it goes, is suited for college work, not everyone needs to spend the money to get a college degree, not everyone needs to spend those 4 years in the cloistered halls of academia to lead successful and fulfilling lives. It is one thing for Norm to say this as a way of recruiting good people to fill jobs in the construction industry and maintain our homes and businesses. It is quite another to make the excuse that a college education is not needed by a significant portion of our population.

We need people who can work with their hands as well as their minds is the argument. We should have more hands-on courses in our high schools, and our community colleges should provide hands-on training for the trades that will prepare our kids to make a “good living” without a bachelor’s degree. This line of reasoning suggests that we have oversold a college degree, that they don’t need the math ability or the reading and writing ability we associate with that bachelor’s degree, that we need an educational system suited for a much broader range of competencies.

As powerful as this argument is, it misses two important points. Despite our general feeling that it is difficult to find a good tradesperson the demand is far less than we would imagine. And while the trades do certainly combine mental and physical work, as they too automate, using more and more powerful technology, the balance is increasingly tipping toward the mental side. The trades are problem solving activities, and it is this problem-solving creativity, whether sitting at a desk or handling a tool at a construction site, that employers are looking for from all trades people, entrepreneurs or employees. They all need the 4Cs skills along with their trade skills.

Won’t Do It!



Socrates (c.470-399)

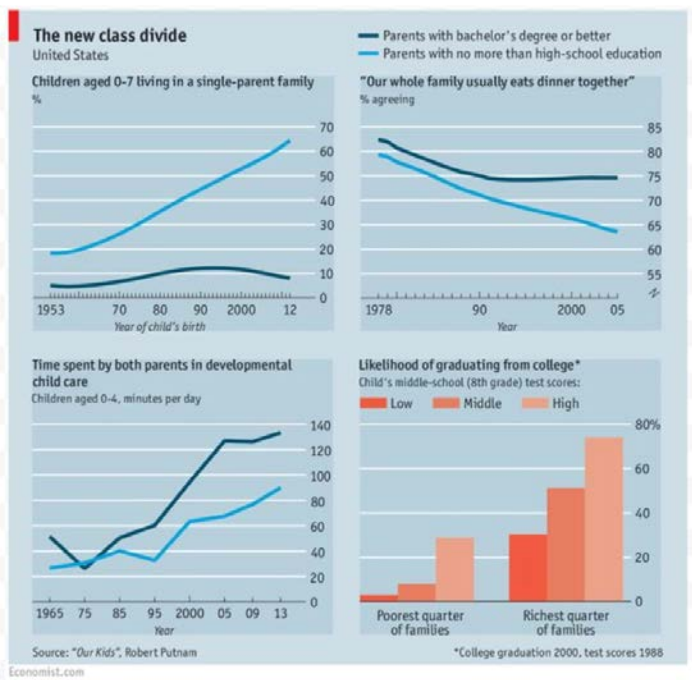
Some 2500 years ago Socrates lamented, “The children now love luxury. They have bad manners, contempt for authority; they show disrespect for elders and love chatter in place of exercise.” Each generation, it seems, claims the next one does not have goals, does not have the work ethic, does not have grit. When I hear this kind of talk about our current generation of young people, I remember Socrates and take it with a grain of salt, for I have trouble thinking that our kids really differ generation to generation so profoundly.

That said, with the rapid pace of technology and the failure of our educational systems to reflect those changes, this lament may well be real today. Our kids know that most of what they are doing and theoretically learning in school these days is irrelevant. Obsolete. They know they will never have to do math on paper when they carry around a computer in their pocket. They know they will never have to memorize facts when they can connect to the Internet and Google anytime. They know they will never have to solve problems like those on the tests that define their life paths without connecting to the Web or to their support group. Even if they cannot articulate the full reasons for their discontent or seeming lack of drive and grit, they know deep down that their classes are just exercises in frustration, Irrelevant!

Imagine instead schooling that uses the great powers in our digital technology to enable students to easily calculate, communicate, learn anytime, anywhere. If we did make our education relevant, if our schooling reflected and taught the skills needed in the digital age both in form and in function, if it met the needs of a flexible and ever-changing world; would our kids still not do it if we made it real? I believe the answer is unequivocal. I don't believe they are spoiled. I don't believe they are lazy. They know a college education is the path to a better life, and in extraordinary numbers it is the path they aspire to.[42]

Given the chance, they will do it.

Our Goal



Our great divide

Our nation is split in two today, segregated into those who will thrive and those who will struggle, those who will follow the upper vector of Robert Putnam's scissors graph and those in the middle class whose lives and futures are not so fortunate, who will likely follow the lower vector with less income, less opportunity, and yes, less happiness. The difference, Putnam tells us, is the bachelor's degree. This degree, that has been the key to entry into the managerial/entrepreneurial class, is today generally required for the jobs that enable families to live their vision of the American middle class. Those who never complete this college degree, those with a high school diploma, those with a community college associate degree, or even those who have taken many college courses, will likely be locked into "repetitive" jobs that we find in assembly-line factories and many service businesses. Repetitive jobs are the ones our K-12 schools were designed to prepare students for. They are rapidly becoming obsolete, disappearing or facing ever-increasing competition from both digital technology and cheaper labor in the "flat world."

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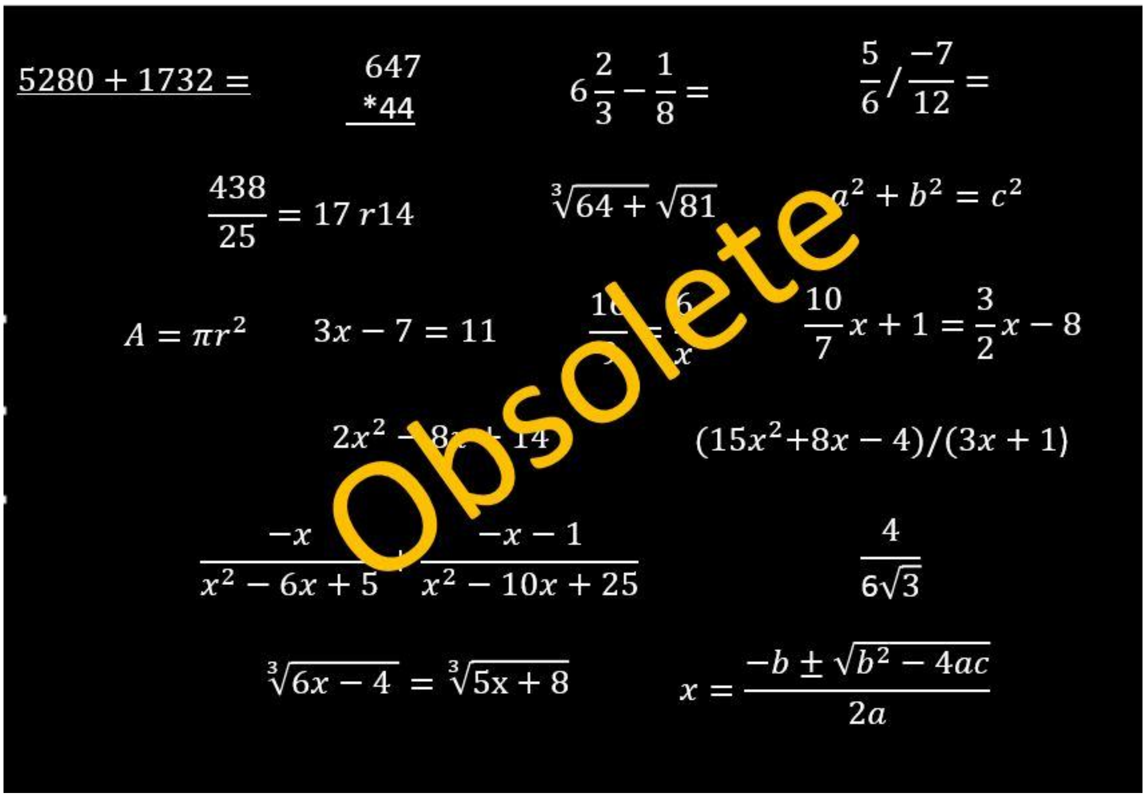
[42] "Eighty-eight percent of 8th graders expect to participate in some form of postsecondary education." Betraying the College Dream – How Disconnected K-12 and Postsecondary Education Systems Undermine Student Aspirations, The Stanford University Bridge Project

If we are to bring our nation together, to agree on common values, to enable all our kids to pursue the happiness our founders envisioned, then it will be necessary for all our kids to have the capability to achieve a bachelor's degree or its digital age equivalent. This is our great challenge. It defines our future.

At a minimum, we will have to double the percentage of students who get a bachelor's degree. To do this we will have to enable 4/5ths of our kids to achieve a new level of learning. To do this we will have to cut the cost of that learning in half to make it affordable by families and governments. To do this we will have to make schools relevant. We cannot push half our middle-class kids out of the American dream, nor can we bankrupt them to achieve it. Fortunately, the educational demands of this new digital age can be met by using digital technology. We can double college success rates at half the cost with learning that is relevant. In what follows, I will define its foundation in actual practice, for the ideas and the tools to meet this great existential challenge and assure our kids can fulfill their dreams already exist.

I will show you how technology, which in every other area of our lives makes us more effective, efficient, and relevant, by increasing productivity, improving quality, and lowering costs can do the same for learning. I will show you how it can make students smarter. I will show you how it can focus learning on the skills needed for the digital age, not on the obsolete skills designed for an industrial age. I will show you how we can cut the costs of schooling in half by joining learning and teaching. I will show you how a simple change in the instructions on national tests can profoundly accelerate educational change. I will show you a new role for teachers and a new style of curriculum based on the model the best graduate schools use to train top managers and creative workers. And I will show you how this reinvention of education can turn kids on to learning as it prepares them for a lifetime of continuous learning–To enable all our kids to thrive.

3. Make Room for the Future



"These Standards define what students need to understand and be able to do in their study of mathematics...Mathematical understanding and procedural skill are equally important..." *Common Core State Standards for Mathematics*

Paper Bag Math

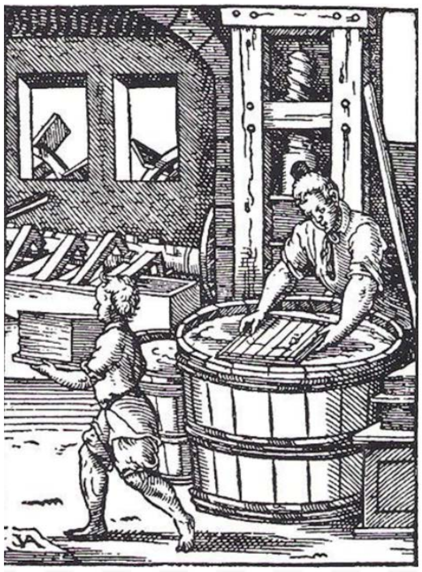
The corner grocery story

As a 6-year-old, I loved to follow my mother and brother to the corner grocery store to pick up what we needed that day for dinner. We would pile the goods on the counter and when complete, the grocer would reach under the counter, pull out a paper bag, lay it flat and start writing down the prices of each of our items in a bold neat penciled list down the face of the bag. He would ask, “Is this everything Mrs. Bardige?” and with a nod make a slash across the bottom and run his pencil up the 1’s column returning quickly to the bottom to write the sum of the 1’s digits, move the pencil in a nearly instantaneous swoop to the top of the 10’s column and write the carry digit there, then swiftly drag the pencil down that column without marking the paper, adding the 10’s digits with the carry and writing the sum of that column below the slash. It seemed to take but a second or two for this trusted grocer to total up and then pile our order into the bag. “Charge it on your store credit?” he would ask, and bag in hand we headed home to make dinner. It made sense in my math classrooms to practice adding columns of numbers. Does it make any sense our digital age?

Today, in business, we would add a column of numbers by listing them on a spreadsheet and using the SUM formula to automatically add them. And if we just wanted a quick check on that work or if we wanted to mentally estimate that sum, we would use a very different method. We would add each complete number, one at a time, going down the list so that we would only have to remember one number. Headmath is not algorithm based, it uses very different methods than handmath. When I subtract in my head, I do not borrow or regroup, I can’t remember those extra digits. Most of us have figured out our own methods of headmath subtraction, and they use approximation and not algorithms. Yet, despite the profound change in our lives from corner groceries to supermarkets and from paper to computers, our students continue to practice paper bag math. They continue to add columns of numbers and subtract by borrowing/regrouping to build fluency they will never need or ever use.

But you might say, this “handmath” practice helps kids understand the concept of addition. It does not; for them addition is a memorization activity, practicing the addition facts. Conceptually, addition when we do headmath, is the process of counting on; 6 + 5 means start with 6 and count on 5 more. That is the way we teach addition in 1st grade, but by 2nd grade we expect students to have memorized the addition table and to do it automatically on paper by place value column, without thinking. They don’t build understanding with this practice, they may only build column addition fluency which they will never use and soon lose.

Paper



Paper introduced to Europe c.1100

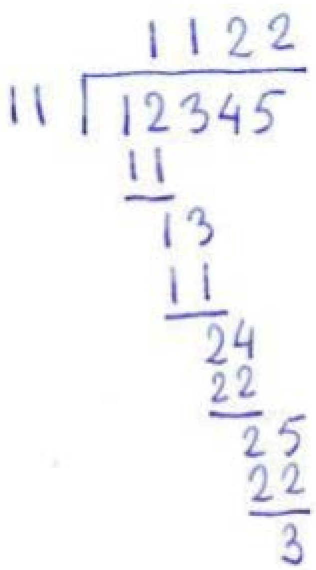
It was invented in China and brought to Europe during the early medieval trade in the 12th century. Expensive and decorated, it was used as a packaging material until medieval craftsmen began to make water powered mills to produce it, soon turning it into a commodity to be used by bookkeepers and eventually printers. By the time I started teaching, paper was still a significant school expense and used rather sparingly. Copy machines were available, but difficult. Mimeograph technology printed black on white in volume, but it was expensive, could be messy, and was confined to school office functions. Teachers used Ditto technology to make paper copies for their students, turning a hand crank, smelling the alcohol, and hoping the wax ink would last long enough to meet their needs. Every year teachers would hoard paper because it always ran out along with the school budget by spring. So, paper was given to students sparingly, cut in half to serve double duty, or students were asked to bring their own.

Slide rules, a technology that required headmath support

With paper a serious expense and copying difficult, teachers made much greater use of oral recitation, in particular of mental math, “headmath”. Classes would practice math facts orally and students would be called on to answer a problem. Workbook practice with the answers running down the side of the page, to make grading easy, meant students rarely had to “show their work” and thus more often rely on headmath to solve problems. Older students, using slide rules to calculate, made headmath a necessity for finding the order of magnitude of the answers. It was no wonder that both teachers and students learned to be much more facile with numbersense. There was time and enthusiasm for headmath before paper-based assessment overwhelmed schools and took over math education.

Today, printers and copy machines along with higher quality paper, that is a much smaller percentage of the school’s budget, have made paper-based handmath the ubiquitous methodology. Even if it is computerized and presented on screen, individualized and automatically checked, it retains its paper-based handmath form and function in both homework and standardized testing. Given our addiction to paper or paper substitutes today, is it any wonder our students can't solve problems in their heads? Is it any wonder they seem to lack the skills most adults associate with learning and using mathematics?

Paper Algorithms



Long division standard paper algorithm

"The only skills long division builds are the skills in using a pencil."[43] Will our kids ever need to use the long division paper algorithm? And I defy anyone to show me how long division builds understanding of the important concept of division. The paper handmath algorithm begins with a headmath guess, then uses multiplication and subtraction and repeats the process again and again to make our guesses more and more closely approximate the answer. While some of the other paper algorithms are a bit more transparent, most are for most kids, totally opaque. They are obscure processes to be memorized, mechanized, hated, and not understood.[44]

It's easy to pick on long division. In its standard form the paper algorithm has to be one of our ugliest. An algorithm is a process. The word, a medieval interpretation the name of the inventor of algebra, the great Muslim mathematician, al Khwarizmi. The word has come into broad use because it is now applied to computer processes, but it has been used since medieval times to describe paper-based mathematical processes. Our kids today have to learn, master, and fluently use paper-based algorithms like two column addition, subtraction with borrowing (regrouping), adding fractions with unlike denominators, solving quadratic equations, or integrating polynomial functions. Some paper algorithms are ugly like long division, and some are more elegant like the chain rule in calculus. Most were developed in their current form to support counting house calculations in the 16th and 17th centuries. They are processes, effectively shortcuts, for rapid and hopefully error-free calculation. They were not designed to make their underlying concepts transparent or understandable.


al Khwarizmi (c.780-850)

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[43] I thank Mike Roberts for this clever retort.

[44] If we want students to practice their addition facts, we should have them do "headmath" addition where they practice mentally adding numbers together, a skill they will use and not practice counting on their fingers to write down an answer on a piece of paper.

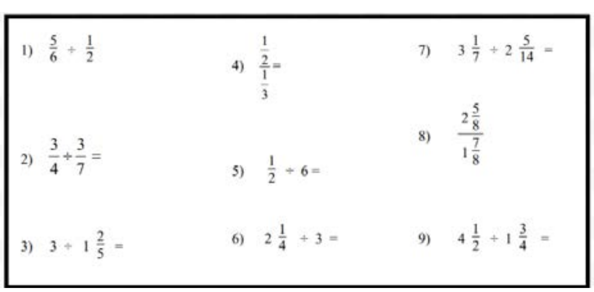
Each of the paper algorithms has two features, a standard step-by-step linear process and a standard visual paper format. Students practice them on a myriad of worksheets and textbook exercises. They also get paper problems in the form of symbolic sentences or equations which they have to translate into algorithmic form to solve. Practicing these algorithms takes up the bulk of the student's school math-time. And it is all a waste! Our students will never use them except for school work or school tests. They will never use them in their business or personal lives because they already carry calculation devices in their pockets. They can always use the Google search box to do most any of the calculations these paper algorithms were designed for. In every case they will do these calculations more quickly, more accurately, and more easily.

Some may worry about worst case scenarios, the total collapse of our electrical system, the castaway on a desert island, or perhaps the random need to get an exact answer to a math problem when all you have is a paper and pencil. I submit there are a lot of other worst-case scenarios I would want our kids to be prepared for that we do not have time to teach in our schools today. But resilience rarely if ever requires exact answers to math problems. Nor is there any value in learning these paper algorithms just-in-case, because if they are not practiced, they are either forgotten or the best case they become rusty slow, error prone, and useless. Either way they have no value. The ability to do headmath, to quickly guess an approximate answer, is much more valuable.

Imagine the time saved by taking the paper algorithm practice out of the school day. Imagine spending that time on building headmath skills, developing visualization skills, understanding concepts, and using math for STEM problem solving and for practicing problem solving skills. We learn what we practice. If we practice paper computation, we learn paper computation. **If we practice problem solving, we learn problem solving!**

If there is one lesson educators should have learned from the past 50 years of adding to the requirements, of making education more and more complex, of pushing more earlier is this fundamental lesson: **Before we add we must subtract.** Before we introduce new demands on our schools, teachers, and students, we have to take some things away. Without that we create an overly-optimized system that fails its primary purpose.

The Problem with Fractions



1) $\frac{5}{6} + \frac{1}{2}$

2) $\frac{3}{4} + \frac{3}{7} =$

3) $3 + 1\frac{2}{5} =$

4) $\frac{\frac{1}{2}}{\frac{1}{3}} =$

5) $\frac{1}{2} + 6 =$

6) $2\frac{1}{4} + 3 =$

7) $3\frac{1}{7} + 2\frac{5}{14} =$

8) $\frac{2\frac{5}{8}}{1\frac{7}{8}}$

9) $4\frac{1}{2} + 1\frac{3}{4} =$

"Early knowledge of fractions and long division predicts long term math success.[45]" says Robert Siegler of Carnegie Mellon University. It comes as no surprise, it confirms our experience. Students' progress in math often break at fractions. Students who were enjoying math, learning arithmetic, and succeeding in

[45] https://www.youtube.com/watch?v= 7YSj0mmjwBM

math suddenly hit a brick wall when they get to fractions. It is a refrain I hear not just from teachers but from adults relating their own math experience. And it is not so much a sudden crash but rather a slow-motion plate tectonic collision. A student may get the idea of a unit fraction as part of a pie but get stuck when asked to compare fractions without unit numerators and with different denominators. The collision worsens when they are asked to find common denominators so they can add unlike fractions, and their math confidence and understanding come to a screeching halt and memorizing takes over when they have to divide fractions by inverting them. All of their intuition, their sense of understanding just seems to collapse and math becomes and remains a black box. So, it should not surprise us, yet it still does, that in the latest NAEP stunning test results *half* of all 8th graders cannot correctly order these three “proper” fractions:

> In which of the following are the three fractions arranged from least to greatest?
>
> $\frac{2}{7}, \frac{1}{2}, \frac{5}{9}$
>
> $\frac{1}{2}, \frac{2}{7}, \frac{5}{9}$
>
> $\frac{1}{2}, \frac{5}{9}, \frac{2}{7}$
>
> $\frac{5}{9}, \frac{1}{2}, \frac{2}{7}$
>
> $\frac{5}{9}, \frac{2}{7}, \frac{1}{2}$

NAEP 8th grade test question

(**2/7, 1/12, 5/9**).

Nor did it fail to surprise me to get emails from Amplify, a well-funded digital learning company, announcing a new math program with its initial content covering fractions. As I read those persistent emails I kept wondering if: “Before we try yet another fancy animation coupled with adaptive drill & kill practice and continuous evaluation, before we go to one more conference talk or Internet website seeking the magic methodology to make fraction concepts and operations transparent to our students, shouldn’t we ask:

Do we need them?

The mathematics education community doesn’t really know why fractions are so difficult for so many kids. We recognize that from the get-go they confuse, for while we treat factions as new kind of number, they are counter-intuitive ones. We can’t count them like whole numbers, for they are really quantities based on two numbers. There are no facts to memorize or practice. Introduced as unit fractions, their value *decreases* as one quantity (the denominator) grows. Unlike whole numbers their values are not unique; there are an infinite number of “equivalent” fractions. Operations don’t work like whole numbers, multiplication shrinks them, and division grows them. Addition/subtraction are more complex processes than multiplication/division; to add and subtract you have to multiply and divide and work with new things called common denominators, **L**east **C**ommon **M**ultiples, and **G**reatest **C**ommon **F**actors. To make fractions even more difficult we represent them visually sometimes as pies, bars, cut up boxes, or numberlines. Yet, despite all this complexity, some kids get them, but so many don’t, and we don’t know why.

It is not for want of trying, for over the past 60 years math educators have sought to meet this challenge with a variety of new curricula, new teaching strategies and new teacher training efforts. Unfortunately, we can show little for our efforts. Siegler suggests the solution is to better train teachers as they do in other countries. But with over 3 million K-12 school teachers and nearly 300,000 full time K-12 math teachers who have about a 10% churn rate, such a massive undertaking is neither feasible or sustainable. And the history of professional development aimed at improving the understanding of fractions bears this out as does a quick glance at math teacher conferences with their majority of presentations on dealing with fractions. Isn’t it time for a new direction that does not keep us banging our heads against that seemingly impenetrable brick wall?

Given such issues, as our schools move from paper to digital technology thus becoming increasingly relevant to the needs of 21st century students, we have to wonder whether students still need to learn confusing paper-based algorithms for operations on fractions? Will they ever need to add, subtract, multiply, and divide messy fractions? Will they ever need to have mastered the algorithm for adding fractions with unlike denominators? And, to push the question even further, in this age in which nearly all calculations are now done using decimals, haven’t we relegated fractions to something akin to Roman Numeral status? We ask again:

- Is it the concept of fraction or just their abstract nature that makes them good predictors of future math success?
- Are students with better visualization skills able to make fraction patterns more concrete and understandable?
- Do some students just have more confidence in their math patternmaking ability because they get fractions which enables them to handle algebra?

I don’t have the answer, but if our kids will rarely, if ever, use them, and if they do not actually build understanding in the long haul, then why learn them in the first place. If we rid the curriculum of fraction paper algorithms, at the very least we gain nearly a year of class time to help students visualize and concretize the mental (headmath) concept of fraction that often proves useful and valuable, or learn other subjects like design, civics, coding, or more science. And perhaps, just perhaps that would be enough for now. But later, I will suggest a more radical solution, that we treat fractions as ratios. For now, though, let's continue to take away.

Mr. Sinaiko’s Lecture

Illustration of Plato’s Allegory of the Cave

Think of it. We all likely spend nearly 20,000 hours sitting in chairs in classrooms listening to teachers, entering into formal discussions led by teachers, or working on teacher assigned exercises. It is an amazing amount of time. And yet if you are like me, there are very, very few of those hours that stick in your mind. Now, I consider myself as having great good fortune in the teachers I had. I can name and clearly remember a half dozen or so who made huge impressions on my life, whose work I try to emulate, and whose caring I try to pass on. But of those wonderful teachers, of that 20,000 hours of seat-time, I remember one hour above all. It was an odd one at that, just a single instance.

In those days, the University of Chicago had its legendary core curriculum, an artifact of the Hutchins' years. It required all its undergraduates, to take so-called College courses, because they were taught out of the undergraduate College as distinct from the Divisions run by the graduate schools, that included two years of humanities. Though I had little experience with classical music, art history, or philosophy, and stayed a hard side physics major; I loved and remember them. They impacted me to a much greater extent than my college major. The classes usually consisted of 3 hours of discussion in "small groups" of about 20 led by an instructor, the University's innovation, and a single large lecture joining all the sections in the University's theater, Mandel Hall.

In Hum 2 we read and discussed the first 6 books of Plato's Republic. The culminating lecture on book VII was given by an assistant professor, Herman Sinaiko, who I had never heard of. I dutifully marched my butt over to Mandel Hall on that Thursday afternoon like any serious student, spiral notebook in hand, to sit through another boring lecture with 300 or 400 other students. But I was soon captivated. The usual shuffling noises, the whispers, and page turning that accompanied all lectures was gone. I was not the only one Herman captured. He took us through Book VII, the wonderful "Allegory of the Cave" for just under an hour. Then Mr.[46] Sinaiko gathered his papers and walked off the stage.


Professor Herman Sinaiko

No one moved. No polite applause. No closing of books or notebooks. No shuffling of feet. Just silence. Everyone in that lecture theatre was mesmerized, stunned would best describe the feeling. At last, in near unison, we stood, applauded fiercely, and finally went onto our next class. I am sure that most of my fellow students, like me, were still vibrating. I do not remember what he said but I remember Plato, and I remember that wondrous feeling of understanding, of learning in the deepest sense of that word. And I remember the meaning of Plato's Allegory of the Cave.

I loved the experience so much that I sought to emulate it in my own teaching, and on good days I found I could pretty consistently mesmerize my own students in the way Herman mesmerized us. After learning to do that for a year, I decided that as wonderful as it made me feel, and as memorable such lectures may have been for my students, that it was about me and not about them, about teaching and not about learning. I came to realize that the best single

[46] Mr. was the standard salutation at the U of C, ostensibly because everyone who taught us had a doctorate and thus there was no need to differentiate.

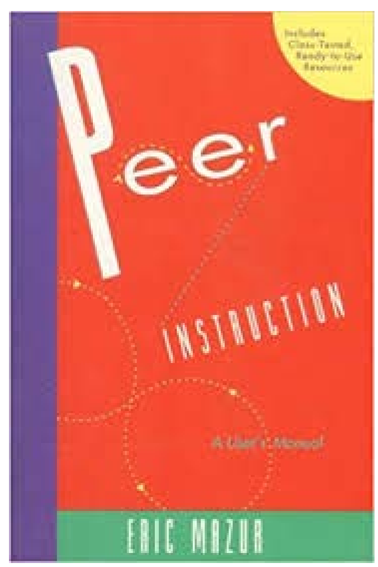
educational experience I can ever remember cannot be the model for schooling. Certainly, it has its value, and I would suggest schools and teachers in the digital age still find those few appropriate opportunities to give great and moving lectures. But education in the digital age is about learning and not about traditional teaching, it is about what students do themselves and do with each other, not what teachers do to students. Mr. Sinaiko's lecture was a profound experience for me, but a one-of-a-kind lesson, one of the very few I remember. They are a reminder to me that this format, as dazzling as it can be, should and must be used sparingly.

Eric Mazur's Peer Instruction

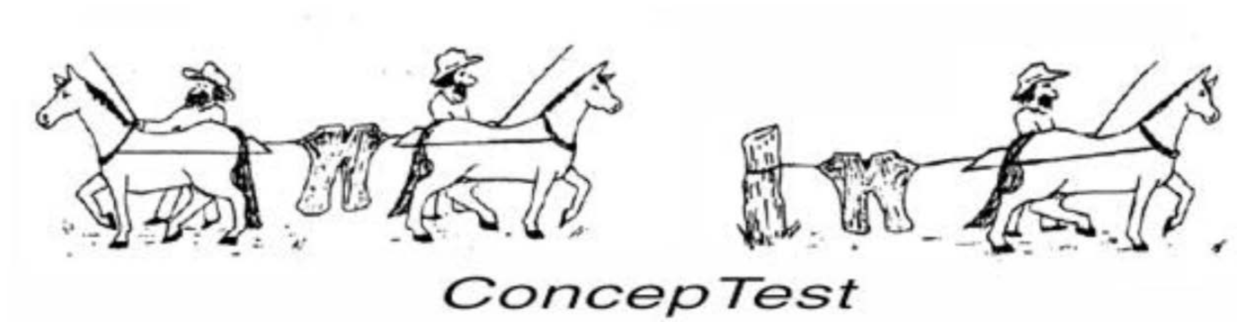
Eric Mazur

I thought I was a pretty hotshot physics teacher until I watched Eric Mazur. Sure, with the right material on a good day I could hold my physics classes in suspended animation. Sure, I loved the subject and conveyed my love and understanding to my students. And sure, I thought my classes were big-time interactive, having learned the art of questioning from Socrates at the feet of his intellectual descendant, Joe Schwab at the University of Chicago. But when I sat in on Eric's Harvard freshman physics course, I soon realized that I was in the minor leagues. And Eric was not even lecturing anymore.

His story is a fascinating one and I recommend watching one or more of his YouTube video talks. A brilliant laser physicist, award winning researcher and teacher, he loved teaching freshman physics to non-physics majors, a challenge few of his colleagues enjoyed. He started out teaching in the style he learned in his native Holland and consistently won student praise and Harvard's top teaching awards. But as he tells it, he went to a physics teachers conference where he sat in a lecture by a physicist he did not know, David Hestenes, who had developed a test he called the Force Concept Inventory. Hestenes found his Arizona State University failed miserably, they may have been able to do traditional physics problems, but they could not answer questions that required them to think conceptually and abstractly to understand the fundamental principles of physics.



Eric initially dismissed the experiment, convinced this had to do with student quality and that his Harvard students would have no problems with the conceptual understanding of Newtonian forces. But soon, his scientific nature got the best of him. He had to try the experiment himself! He was crushed to find that his students also failed miserably. They had not understood the fundamental physical concepts. Like any good physicist, after considering all the variables, Eric recognized that the model was broken. His lectures, no matter how beautifully crafted, were not helping students build their fundamental understanding. Lectures, no matter how wonderful or how well prepared, did not enable students to learn the concepts.



The Levi Straus trademark shows two horses trying to pull apart a pair of pants. Suppose Levi had only one horse and attached the other side of the pants to a fencepost. Using only one horse would: (a) cut the tension on the pants by one-half, (b) not change the tension on the pants at all, (c) double the tension on the pants?

He constructed a new model of education that he called Peer Instruction whose fundamental premise was that students should learn physics by teaching each other. And he developed a comprehensive new methodology using ConcepTests[47] to engage students and focus them on the concepts and not the superficial calculations. He wrote multiple-choice questions, used handheld devices to share results, and developed a process to engage students. The results, as you would expect from a fine scientist, have been well-researched and outstanding. If the best physics teacher I have ever known, an elegant and amazing lecturer, has given up the lecture as his primary form of teaching, the rest of us should reevaluate what we think our lectures are giving to our students, and use technology, as he used it in a relatively primitive fashion, to enable students to learn themselves and from each other instead of from the lecturer. Are you ready to give up the lecture as your principal form of teaching?

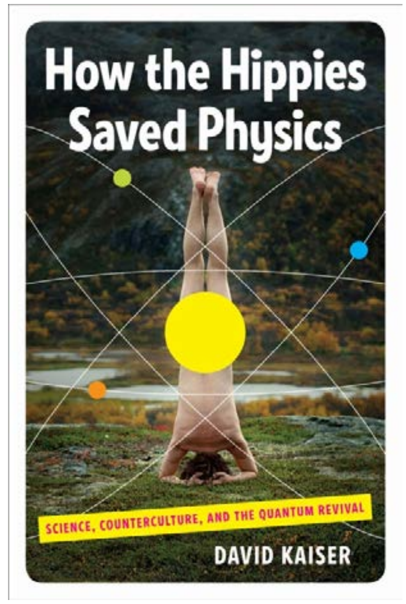
Textbooks and MOOCs

When MOOCs were the rage in higher education, I asked David Kaiser, a physicist, brilliant author, and professor of the history of science at MIT, when he was going to do a MOOC. Dave has won teaching awards at MIT and writes fascinating books on the history of physics[48]. Who better to

[47] *Harvard Journal*, Summer 1995 by Kathleen Koman
[48] I highly recommend *How the Hippies Saved Physics.*

do a MOOC or bring his wonderful style of teaching and presenting important physical ideas to more people? But he was not at all interested, and as far as I can tell several years later, has not done any.

“Why” I asked. “Because you can’t change them,” he replied. As he explained, one of the most wonderful aspects of teaching a course year after year for a great teacher is the opportunity, indeed the necessity, to change and adapt the course in general and the presentations in particular. His reaction brought back a vivid memory of my first couple of years of teaching high school physics. I usually carefully prepared my lectures which were the standard fare for most of my classes.

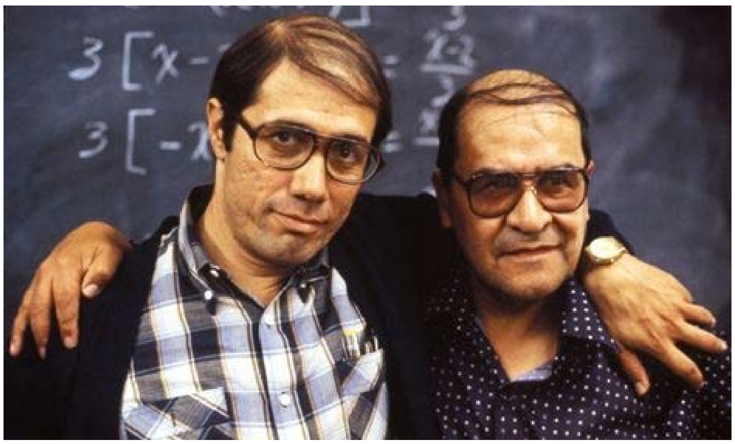


Occasionally too busy, too tired, or too lazy to develop a new one, I would grab my lecture notes from the previous year which I thought pretty good. The class usually started all right, but I soon got into trouble. The coherence was gone, the presentation no longer seemed to make sense to me. I don’t know if my students realized that I was stumbling, they were too busy taking notes, but I did. So, I would stop lecturing, tell my class what I had done, apologize, give them time to work on their assignments, and come back the next day with a fresh lecture. One of the things that makes teaching such an interesting job is the year-to-year, day-to-day, and even student-to-student opportunity for improvement, for growth, for learning. For good teaching is an evolutionary process.

This has not been true of curriculum. MOOCs like textbooks are expensive to produce. They are linear, moving from topic to topic in a standard form, a continuous line of lesson following lesson. They are thus difficult, often impossible, to update or change. Once created, except for minor revisions, they are for all practical purposes – fixed. Yet, the world is constantly changing, and even more importantly students are constantly changing. A fixed curriculum or presentation cannot work. It will no longer work to expect textbooks to have a 7-year lifespan. Nor will MOOCs, made once and used again and again, work either. The analog continuous linear sequence of lessons that represent a course is no longer functional in the digital world.

The digital world is discrete. It needs education to be built in small packages, flexible, easy to change, constantly renewing and growing. The metaphor for the analog age and the MOOC is the book, done once and then published in at best yearly versions. The metaphor for digital age educational content is the newspaper, renewed and reimagined every day. One is fixed, unchanging, the other flexible and constantly refreshed. One is designed to be the same for all students, the other can be different, personalized, to suit the needs and interests of each individual student. One is designed to feed students information, segregating knowledge from practice, the other integrates knowledge and practice.

"Stand and Deliver"



It was an appropriate title for the movie[49] about Jaime Escalante, and it is an appropriate title for the role that teachers continue to play. We all too frequently see our role in both K-12 and in college as an actor standing and delivering. As problematic as that vision may be for our physical classrooms today, it is even more of a problem for digital learning classrooms. It is the reason that the most common refrains we hear about teaching online is how much harder it is, how much more time it takes, how difficult it is to stay connected with students. For we have taken the "stand and deliver" classroom model and transmogrified it into the online model.

Stand and deliver teaching puts the educational burden on the teacher. Students are the recipients of the knowledge in the head of the teacher. I am reminded of this old Egyptian image of Pharaoh Akhenaten's sun god reaching out to him handing him his gifts. In the paper classroom, the teacher's ability to motivate, to tell a story, to organize, and to simplify the textbook's knowledge, when that was nearly all the content available to students, stand

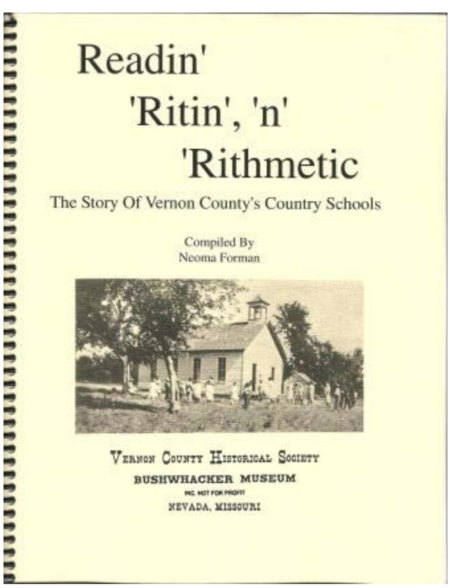
[49] *Stand and Deliver* with Edward James Olmos

and deliver was a reasonably efficient way to narrow the spigot and enable the student to absorb the content. Eye contact, proximity, raised hand signals, and easy verbal interaction made this model sufficiently flexible, engaging, and rewarding.

But stand and deliver in the digital classroom without eye contact, proximity, or easily recognized hand signals requires us to rely on other means to hold the engagement of students or to recognize their learning signals. A number of tweaks to the lecture have been tried to make the model work. MOOCs make their video lectures less than 7 minutes long and separate them with student activities. Teachers make themselves available 24/7 to talk online with individual students. New communication and presentation formats have been tried to enable teachers and students to engage with each other. Teachers have to provide a wide variety of additional materials and different formats to accommodate student needs. But if technology is to enable a more effective and efficient learning experience then we must think anew about its use and let go of the thousand-year-old stand and deliver tradition.

The Web has a different just-in-time delivery model than the lecture. It gives students a wide variety of opportunities to get what they need to learn, whether it is a short video description, some pertinent information, a dynamic interactive experience, or connections with others who can help. Stand and deliver worked because it was just-in-time delivery and often where a student could ask the teacher a question and get an immediate response. The Web becomes a better educational tool when it is likewise used as a just-in-time repository.

When the 4Cs replace the 3Rs



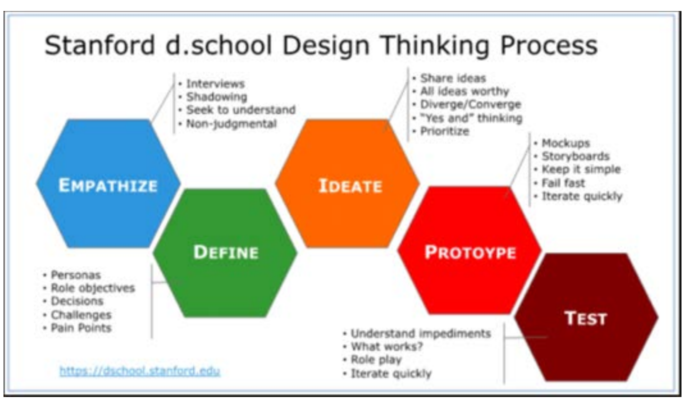
The 3Rs have been the backbone, the bedrock of American education for going on 2 centuries. They have been the purpose of education and their tests have set the goals of a quality education. Read, Write, Calculate, (Readin', 'Ritin','n' 'Rithmetic) have been our shorthand for school success. But today, the business world has defined a new set of goals for success, a new set of basic skills to replace: "Can you read?", "Can you write?" "Can you calculate?" They are called the 4Cs – **C**ritical thinking, **C**reativity, **C**ommunication, and **C**ollaboration. Business wants to hire people who can think critically, creatively, collaboratively, and communicate to solve novel problems.

While they may think they are looking for people who can read, write, and calculate, business values the 21st century skills more highly, and leads us to ask, not whether students can read, write, and calculate, but what can they read, what can they write, and what can they calculate? Today, we use a blanket measure, the same for all our kids. We simply expect students to be able to read anything, especially huge textbooks, to write long coherent and well-argued papers, and to calculate mainly on paper any arithmetic and basic algebra problem. But in this digital age, communication takes a variety of forms, especially visual forms. **Being a good communicator does not *require* you to be a good writer.**

In this digital age, reading measured in the ability to consume, remember, and regurgitate large books is not nearly as important as being able to find and evaluate information when you need it, in any media form, in other words to think critically. In this digital age, working with others, pooling resources, collaborating is a key skill that replaces the extraordinary reliance on individual production we currently insist on. In this digital age, calculation is a machine function; problem-solving is the critical skill and we do that by finding, following, and using links on the Web. And in this digital age, since machines solve repetitive problems, humans must be able to solve the novel ones, which means that creativity is the most highly valued skill of all.

We are not so much replacing the 3Rs with the 4Cs as we are subsuming them. The 3Rs are no longer general skills that we seek for every student. Instead we ask, read what, write what, calculate what, and focus our prime attention on the digital age 4Cs. The digital age overwhelms us with data which is why many call it the "information" age. It is our job as educators to help our students deal with this abundance and to use it to solve the problems they will naturally and constantly face. This is the reason we now must prepare at least 80% of our students to have managerial skills; to, in the full meaning of the word, manage information and a world with rapidly changing demands and opportunities. These 4Cs, at a minimum,[50] do define the foundational skills all students need as required by business today.

Critical Thinking is Problem Solving



We are called as PK-12 teachers to ask, "Why is problem solving so hard to teach?" As problem-based-learning, it is highly successful in the top graduate schools of business, medicine, and law using case studies. As project-based-learning, it is the key ingredient in a handful of American high schools pushing the envelope. As authentic problems, it has been the dream of educators for more than a century. As Socratic questioning and critical thinking, it remains the pedagogy of elite colleges like the one I went to. And as deep learning it is the dream of AI as well as leading educators. And

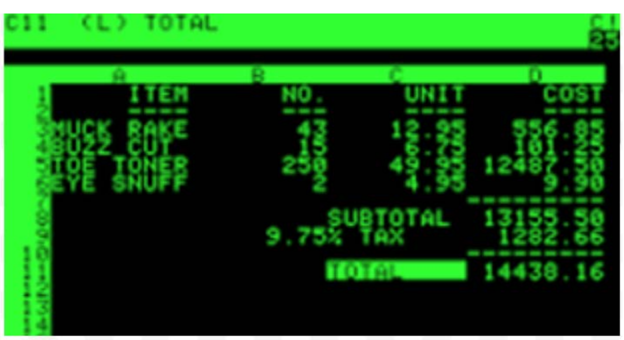
[50] For you may, as others have done, want to add C-words like Confidence or Captaining.

yet, in most of our schools, it remains just a bunch of overly simplified elusive exercises that are found in the problem-solving section of textbook assignments in math, science, and other subjects. Those traditional story problems only pretend to be real problem-solving, yet most of our kids find them stumblestones. We make excuses pretending that smarter problems, better reading skills, or procedures called by an acronym will enable kids to be successful thinkers. None have worked.

What if we look at the problem of problem-solving as a technology issue? Problems that feel real, that kids would likely find interesting, that are worth working on, are rarely paper and pencil problems. Even the simplest problem exercises that have the taste of authenticity today require some serious computation or research. And the real problems, the problems our kids can find worth the effort, certainly require substantial computation and research. With spreadsheets and the Web as primary tools, we can finally ask students to solve problems that they will find relevant and take pride in solving, problems that are worth their grit. So maybe, just maybe, the problem with problem solving is that the problems have not been difficult enough.

When we give our kids the tools of today's adult problem solvers, spreadsheets for computation and the Web for data, collaboration, and research; we can, without much difficulty, make problems authentic and interesting. We can give students the tools for working on them, and the powerful processes they can learn to use for problem solving like *design thinking* and *functional thinking* (which we will look at in detail soon) to enable them to practice problem-solving in a standard, replicable, and most importantly in a creative way. Problem solving no longer need be a boring process where reading and calculating are major roadblocks to success, but a creative process that can be collaborative and fun. Such rich problems that demand technology to even consider them, automatically open students to learning critical thinking, to use design thinking to see the problem in its context, consider the nature of the data, build a model, iterate the model, and ask the creative question, "What if…"

Creativity



VisiCalc 1979 screen

Allen Sneider was likely the first accountant to see or work with *VisiCalc*, the first spreadsheet. We saw it at a meeting of the New England Apple Tree (NEAT), the Apple users club in the Boston area.[51] Dan Bricklin and Bob Frankston had come to show a beta version, and he was immediately taken by it. An accountant in one of the Big 7 accounting firms in Boston, he was well versed in building business models for clients. He describes taping standard 11 by 14 accountant

[51] I was a member of NEAT, and I remember that first VisiCalc presentation.

worksheets together spread across the big conference room table. Number 2 pencils and art gum erasers were the only writing instruments allowed in the room. His assistants had filled the worksheets with numbers, and stapled small rolls of 2" paper tape from adding machines that "justified" the calculations. They were building models of new products, real estate ventures, or potential business acquisitions. When finished, Allen would bring his clients into the conference room to feast on the masterpiece, go through the model step-by-step, and comprehend its conclusion.

After his orientation and time to contemplate the results, the client would almost always ask a painful "what if" question: "What if the interest rate went up to 6.5%?" "What if we change the price point to…?" With the client's "What if's…" in hand, Allen and his team would, like an architect designing a building, erase and recalculate to rebuild the model at significant work and substantial expense. "They rarely did it more than once or twice," Allen said, "it was just too costly."

When he saw VisiCalc for that first time, he immediately grasped its potential for asking "What if…" as often as he or the client wanted. It gave the model creative wonderful power. This creative power to build models ask, "What if…" and then iterate the model and ask, "What if…" again, iterate and ask again, and again… has made spreadsheets the most important technology in the workplace.

Today, I can't think of a job where this kind of creativity doesn't play a role. "What if…" has become central to costing out a job, to defining what is and what is not to be in a product or service, to deciding how an organization can meet new challenges. It is the story of business, of work, and of life in the 21st century. It is the method of science, build a model and ask "What if…" It is central to the arts, where we try to make patterns and ask again and again, "What if I do this to the pattern?" **If we really want to prepare our kids for the digital age, then we must get them practicing creativity, practicing asking "What if…**

Communication

My mother had the sunniest and most easy-going personality of anyone I ever met. Few things riled her, usually a fight between my 16-month younger brother Steve and me, but just as often it was the comic book I was quietly reading. "Arthur," she would yell, "You are never going to learn to read!" She was in great fear that my ability to read would be permanently stunted by the Superman, Batman, and Wonder Woman comics I 'read' and reread from the 3-foot- stack I protected in the bedroom I shared with my brother. She likely had reason for such fear, for other than the Hardy Boys and Tom Swift books that I was reading by age 11 or 12, I had shown no interest in reading a 'real book' in school or out. My brother was reading more than I was, and I am sure my mother found my reading habits quite distressing and an object of conversation with friends and family.

I will tell you one of my life's secrets; I feel guilty to this day for the trauma I put her through since my ability to read ceased to be an issue long ago. Far from stunting my reading skill, those comics may well have contributed to the development of another skill, visualization, which has in fact dominated most of my life and my work. While comic books play a much smaller role in today's world then it did when I was growing up, I think I could have made a much better argument for having a proclivity for them. In the 20th century as in the 19th, text was more important than graphics for telling most of the stories we had to learn and to tell. There were comic books, movies, and visual magazines like Life and Look, but most of our communication was text-based. Images in our work, schools, and even play were supplements to text, enhancements of the message or story being communicated. Today, it is the opposite. In our digital age, text supports images in our communication. Graphics in the form of icons, drawings, symbols, emoji, photos, and even videos are the main message on the web, in our social media, on our websites, or our posts. Text plays a diminishing supportive role.

This profound change in significance between text and images in our work and daily lives leads us to ask whether education needs to change its ELA emphasis. Should we not today focus on visual communication, design, images, multimedia and less on text. This is not to suggest we no longer need to teach our children to read, but rather that the high levels of text skills in both reading and writing we currently require should not be the criterion for college readiness, college learning, or workplace success.

Instead, should we now focus on the larger vision of communication as a fundamental digital age skill to **give every student the opportunity to communicate in the ways they are most comfortable with, for in this digital age different forms of communication are valuable**. We should no longer link intelligence and school success to reading text. Communication–oral, visual, kinesthetic, verbal, or musical, in all its rich and varied forms is the key 21st century digital age skill. It is this skill that our kids have to practice. It is this skill that our kids should be judged by. It is this skill we must treasure. I hope my mother knows I was just ahead of my time.

Collaboration

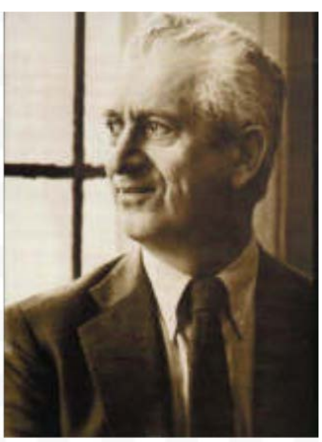
Google London headquarters designed to enhance collaboration

Collaboration, perhaps the most important 21st century skill is very often considered by our schools today, cheating. Students caught talking to each other exams or looking at another student's paper are either yelled at or disciplined for cheating. Homework is supposed to be an individual activity and students are punished for cheating if their paper resembles another. In English or Social Studies, if you are caught copying something or someone without attribution, then you are plagiarizing and treated as if you have committed a crime, cheating. And if you are doing a project with a group of students, be sure your work and effort are your own not the work of others or you are cheating. We are training students from the earliest school age to work individually, to “do your own work,” to not cheat. We are still teaching our students to be rugged individualists, independent, self-motivated, and self-reliant. We are teaching 19th century skills.

Today, collaboration is one of the 4Cs skills because it is seen, in survey after survey of business, to be critical to digital age problem solving. Creative problem solving is considered a group activity today, and business would no more consider isolating individuals in the workplace than taking them off the Internet. Offices and universities are designed to breakdown silos, to have courtyards and corridors, like the design of the new Google headquarters in London, where people can constantly meet, share ideas, and engage in group problem solving. The best employees are considered the ones who work well in teams, who are good collaborators.

Yet, we educators act as if collaboration is either a skill we are born with and need not practice, or one we magically gain when we require it. Despite the importance given to teamwork and collaboration in sports, we still do not consider it a skill we should learn in school, or practice in school, a skill no different from reading or numbersense. Learning to collaborate in school, if it is to be a central mission, requires us to rethink education from the ground up. But even if we are not ready to take on that big task, we can start by making our classrooms meeting places where silos are not just torn down between subjects but torn down between students, where students are supported and encouraged to learn to collaborate. Where collaboration is not cheating!

Bloom’s Problem Solving Vision



Ben Bloom

I would never have guessed when I first saw Ben Bloom walking the corridors of Judd Hall some 55 years ago looking more accountant than college professor, and heard him lecture on what is now labelled as his Taxonomy, that it would become so central to education and so long lived in an arena that tends to chew up new ideas and recycle old ones. Bloom’s work has achieved a remarkable status as a set of core principles that have flexibly changed with schooling fashions.

Bloom’s Taxonomy, as it has come to be called, was not a theoretical work, but rather a practical pattern developed by a series of committees to organize the objectives of schooling in the cognitive domain into 6 levels.[52] Following the old maxim “what is tested is what is learned,” the Taxonomy is focused on

Lower Order	Higher Order
Knowledge	Analysis
Comprehension	Synthesis
Application	Evaluation

Bloom’s Original Taxonomy

[52] The Original Taxonomy

improving tests. The committee found that for most teachers' lessons and test questions were on the "lower order thinking skills" particularly on what the Taxonomy labeled as Knowledge, the teaching and assessing of factual information. The "higher order thinking skills" (analysis, synthesis, evaluation) while the most important and valuable for problem solving, were rarely taught or tested.

His view of problem solving remains relevant today:

> *… common observation would indicate that individuals in general tend to avoid real problems solving. When presented with problems, they usually apply a limited stock of techniques to them and are frequently satisfied if a partial solution is obtained. If the techniques do not work, there is a strong tendency either to reorder the problem completely (that is, to make a new problem) or to escape from it entirely. Rarely do individuals stay with a difficult problem for any considerable length of time and try increasingly varied procedures for attacking it. Yet, we need more than ever to help students develop problem-solving methods which will yield more complete and adequate solutions in a wide range of problem situations. It is hoped that the taxonomy's analysis of this area will facilitate the exploration of new methods of teaching for high-level problem solving and assist in evaluating these methods.*[53]

Yet, despite widespread acceptance and recognition of the importance of Bloom's Taxonomy and the widespread belief in the importance of problem solving,[54] it has barely impacted our educational system which remains focused on "knowledge", on students gaining and using information. As long as knowledge is considered a scarce commodity, as long as it is relatively hard to come by, as long as it requires a teacher to transmit, then it will be considered the foundation for higher order skills and crowd them out.

But in the digital age, knowledge is no longer scarce, no longer hard to find. Being readily available, all but a small nugget (like the multiplication facts) need be memorized. It is the skill of finding knowledge on the Web that has become foundational. This skill requires not only the ability to search, but the critical thinking ability to differentiate. It requires not only the ability to google, but the ability to communicate both to the search engine and to people, what you are looking for. And it requires creative and collaborative skills because the knowledge being found can and should be shared and used. If the lower order skills are by-products of search, then in this new digital age our kids and teachers can concentrate on the higher order skills and use the powerful new tools of the digital world to learn those skills.

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[53] Bloom, et al, *The Taxonomy of the Cognitive Domain* Pg. 42-43

[54] Including in the Common Core

Over the Rainbow

"Over the Rainbow" by Harold Arlen and E. Y. Harburg is considered the greatest song of the century and the greatest song written for a movie of all time, it is the love song in "You've Got Mail", and it is my very favorite song. I never tire of listening to it. "Yip" Harburg not only wrote beautiful love songs, he wrote all of the lyrics for "The Wizard of Oz", "Brother Can You Spare a Dime" and the musical "Finian's Rainbow." He left us with music that beautifully expressed his deep belief in and search for an equitable society.

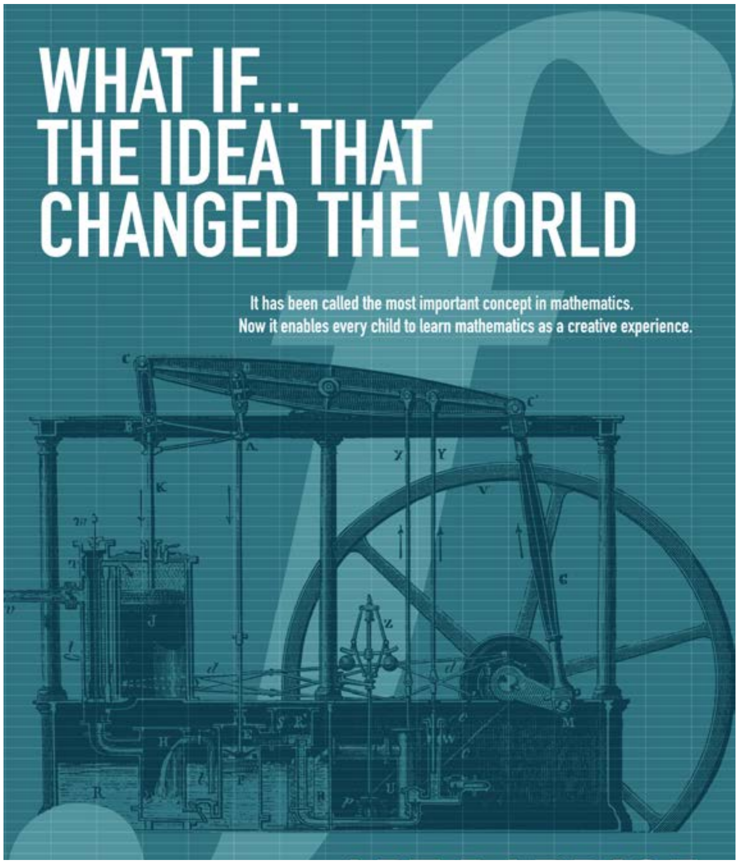
Today, if he were alive, Yip Harburg might well be writing lyrics about math. For unlikely as it may seem, mathematics has become the main driver of inequity in our nation. It is the foremost academic reason students do not complete a college education. We can no longer claim that math is not a necessity for everyone. We can no longer claim that it is okay for people to not get math, okay because in the arts and trades they likely won't need it, okay because machines will calculate for us. We can no longer claim that the math we expect our children to master should be determined by college math faculty who see their job as producing qualified mathematicians and not students prepared for the digital age. **Math has become central to not just work and life, but to our thought processes as well. It is not only key to good STEM jobs, it is required for the managerial tasks everyone will need to perform in the 21st century. We can no longer dismiss math as a nice to have.**

Over the past quarter century, a small movement in math education has been slowly spreading. It is called Quantitative Reasoning[55]. It is not yet cohesive or coherent, but it represents the efforts by math education leaders to make math about real world problem solving. It focuses conceptually on making units (dimensions) a critical part of problem solving. As my friend Corri Taylor, professor of economics at Wellesley College and one of the leaders and pioneers in this movement tells her students, "Let the units be your guide."

It is beginning and we must follow it to ask, "What mathematics is essential for life and work in the 21st century?" And how can we enable every student to develop the numbersense, the unit sense, the understanding, and the fluency they will need to reason quantitatively and become good digital age problem solvers. I believe this to be the most important question education faces, for solving it will truly enable every student to "fly over the rainbow."

[55] Sometimes called Numeracy or Quantitative Literacy, or just QR.

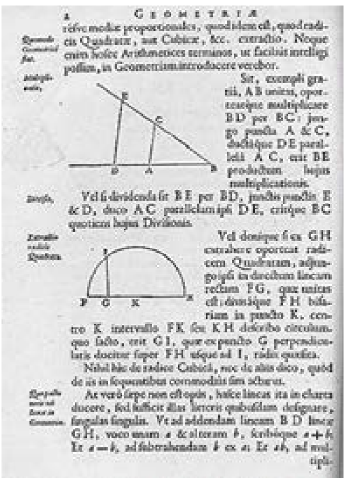
4. The Idea that Changed the World



It is called the most important concept in mathematics. It birthed the scientific revolution. It enabled computer programming. And its latest incarnation, spreadsheets, has revolutionized business. Yet, few of us can name it; still fewer of us can define it; and it is unlikely that even a handful of us know its evolution. This idea, that changed the world will now enable every student to learn mathematics as a creative experience.

Functional Thinking

The wonder of it has to be the timing. Within just five years, between 1635 and 1640, five men, living in different places, with little connection or mobility, each building from his own foundation and focus, produced a revolutionary way of thinking that transformed mathematics and the sciences, fostering our modern world. Each grappled with the concept of variable and with the relationships between variables which later defined functions.


2 GEOMETRIÆ

résve mediæ proportionales, quod idem est, quod radicis Quadratæ, aut Cubicæ, &c. extractio. Neque enim hosce Arithmetices terminos, ut facilius intelligi possim, in Geometriam introducere verebor.

Sit, exempli gratiâ, AB unitas, oporteatque multiplicare BD per BC: jungo puncta A & C, ducoque DE parallelâ AC, erit BE productum hujus multiplicationis.

Vel si dividenda sit BE per BD, junctis punctis E & D, duco AC parallelam ipsi DE, eritque BC quotiens hujus Divisionis.

Vel denique si ex GH extrahere oporteat radicem Quadratam, adjungo ipsi in directum lineam rectam FG, quæ unitas est; dividoque FH bifariam in puncto K, centro K intervallo FK seu KH describo circulum, quo facto, erit GI, quæ ex puncto G perpendicularis ducitur super FH usque ad I, radix quæsita.

Nihil hîc de radice Cubicâ, nec de aliis dico, quòd de iis in sequentibus commodiùs sim acturus.

At verò sæpe non est opus, hasce lineas ita in charta ducere, sed sufficit illas literis quibusdam designare, singulas singulis. Ut ad addendam lineam BD lineæ GH, voco unam a & alteram b, scriboque a + b. Et a − b, ad subtrahendam b ex a. Et ab, ad multipli-

1637: Rene Descartes, living in the Netherlands, published *La Géométrie,* as an appendix to his great philosophical treatise *Discours de la méthode,* that joined algebra and geometry. He invented a new kind of quantity to represent lines and not numbers. Though he called it a new geometry because it dealt with lines and shapes, it was a new kind of algebra, an algebra of lines[56] represented by variables and not points represented by numbers. In this second page of *La Géométrie* he begins to meld algebra and geometry by showing us how, not only to add and subtract lines, but to multiply and divide them. We may say of this brilliant work: before Descartes people saw lines as geometric constructions labeled by their endpoints. After Descartes they conceived of lines as representations of the relationship between two or more variables, labeled by them.

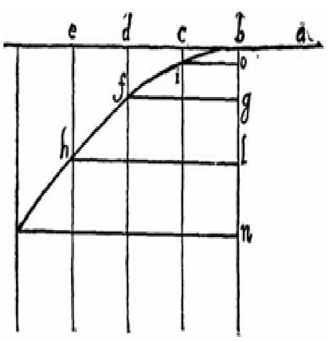
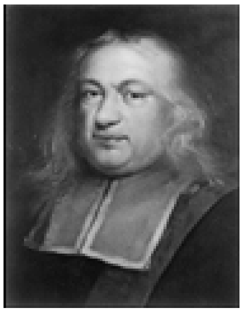


Fig. 108

1638: Galileo Galilei, age 74, nearly blind, and under house arrest in Florence for advocating Copernican heliocentrism, secretly sent a new manuscript to Holland for printing. It was his great work *Dialog Concerning Two New Sciences* written in Plato's Socratic style as conversations between Galileo the teacher, an Aristotelian, and a wise man. The first new science dealt with the mechanical properties of matter. The second, still taught to every school child, was the study of motion. In its fourth and final dialog, Galileo focused on the motion of projectiles and showed that their seemingly complex path could be understood as the composition of two simpler motions, one horizontal the other vertical. The horizontal motion in the absence of air resistance was "uniform," constant with time, the vertical motion because of gravity was "accelerated" varying with time as he had shown 40 years earlier with his inclined plane experiments, the distance covered by a falling body during each unit of time increased as the square of the time. "…if we take equal time-intervals of any size whatever, and if we imagine the particle to be carried by a similar compound motion, the positions of

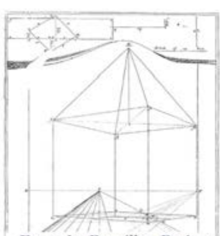
[56] Effectively a collection of points.

this particle, at the ends of these time-intervals, will lie on one and the same parabola."[57] A line can represent the motion of a body using time as a variable.[58] **This amazing image of projectile motion from that dialog, the first conceptual graph[59] ever published, shows the parabola composed of horizontal and vertical motions.**



c**.1638**: **Pierre de Fermat**, often remembered today for his cryptic marginal note known as "Fermat's Last Theorem," studied curves, developing ways to find their maxima, minima, tangents, points of inflection, and rates of change. His work, the precursor to the differential calculus of Newton and Leibniz, includes many of the topics students study in school today in college algebra, precalculus, and differential calculus courses. He spread these ideas from 1635-1640 through letters to friends and colleagues from his home in Toulouse, France.

c**.1635: Bonaventura Cavalieri**, Galileo's protégé, studied areas and volumes and sought to find general ways to compute them. The first to find the area under a parabola, his work is considered a critical prelude to integral calculus, with his "principle of indivisibles," imagined shapes built by composition of very small incremental slices. "Cavalieri asserted that a line was made up of an infinite number of points (each without magnitude), a surface of an infinite number of lines (each without breadth), and a volume of an infinite number of surfaces (each without thickness)."[60]



From the Brouillon Project
Geometry of an Art.

1639: Gerard Desargues published his study of the geometry of projected shapes introducing mathematical transformation and invariance. Though considered brilliant by his contemporaries, Desargues projective geometry did not play a significant role in mathematics until the 19th century but represented another facet of this new way of thinking about relationships between lines (between variables).

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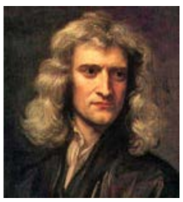
[57] http://oll.libertyfund.org/titles/753

[58] I am indebted to Professor Owen Gingerich for this insight.

[59] There were earlier images that we could view as graphs, perhaps charts would be a better word for them, that for example traced the path of a planet over time. (See Bruce S. Eastwood, Astronomy and Optics from Pliny to Descartes, Wariorum Reprints, London 1989, p. 278). But I do not consider these to be conceptual representations.

[60] W. W. Rouse Ball, *A Short Account of the History of Mathematics*, 1888

I cannot account for the amazing mathematical and scientific generativity of the early 17^{th} century as the random arrival of individual geniuses, remarkable as they were. Nor can I ascribe it simply to a loosening of the reins of religion and other institutional structures. It must be recognized as a revolution enabled and afforded by a new and more general way of thinking, a shared way of thinking we now call functional or "What if…" thinking. It was based on a new kind of object defined by Descartes, a variable he labeled "**x**", that represented a line and not just a number, that could be linked to a formula, a table of values, or a graph. It provided the sciences ways to build complex systems from simple building blocks through composition as Galileo and Desargues had. It enabled mathematicians to ask questions about change as Fermat or Cavalieri had. It would enable natural philosophers to ask the experimental science question, "What if…" It gave artists and scientists ways to visualize change as transformations. Newton described the simplicity of this wonderful new way of thinking in 1671:



I am amazed that it has occurred to no one…to fit the doctrine recently established for decimal numbers to variables, especially since the way is then open to more striking consequences. For since this doctrine in species has the same relationship to Algebra that the doctrine of decimal numbers has to common Arithmetic, its operations, of Addition, Subtraction, Multiplication, Division, and Root extraction may be easily learnt from the latter's.[61]

Over the following 40 years, a tenth of the time from *Liber abbaci* to *La Géométrie*, the world of mathematics and science exploded. Newton and Leibniz developed calculus with general rules for finding the rate of change of curves and the area beneath them. Leibniz applied the word functions to describe the building blocks of curves.[62] Newton applied these ideas to motion developing the first true cause-and-effect based physical theory and thus defining the course not only of physics but of the entire scientific revolution. Euler the greatest of the 18th century mathematicians gave functions its distinct symbol. And from these humble beginning, function has now become, in the words of the Chair of Harvard University's math department:

> *Perhaps the most important concept of mathematics is that of function, which provides us with the means to study dependence and change.*[63]

Functions and functional thinking are today at the very center of our world, pervading the disciplines, those we call STEM or STEAM[64] as well as those we label the "Liberal Arts" to include the whole of the humanities.

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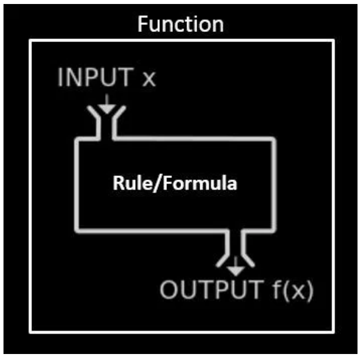
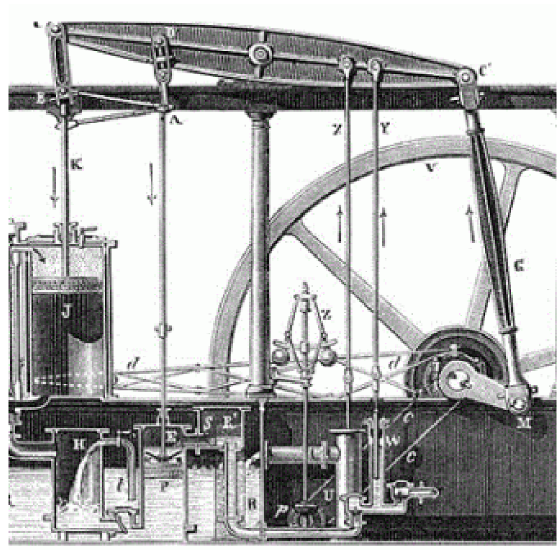
61 By Isaac Newton in *A Beautiful Question: Finding Nature's Deep Design* by Frank Wilczek
62 Jere Confrey, David Dennis, Functions of a Curve: Leibniz's Original Notion of Functions and its Meaning for the Parabola, http://poncelet.math.nthu.edu.tw/disk5/js/linkage/m7.pdf
63 Peter Kronheimer, Chair of Mathematics at Harvard posted in the Harvard student handbook
64 Science, Technology, Engineering, Art, Math

Machines are Functions

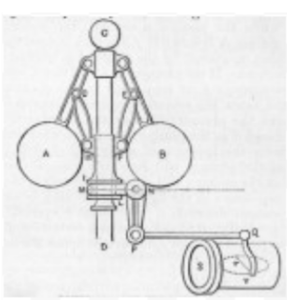


I do not know who, when, or where this iconic mathematical representation of function was developed. It is, however, one of the most powerful and ubiquitous of all mathematical images, and I think the most important. It is taught to 2nd graders and used by STEAM professionals. It is called a function machine, and it represents the way we think about change, cause and effect, and technology as well as mathematical functions. For since the dawn of the industrial age we have pictured our world as a machine, as a function, a "rule" that converts and connects an input to an output.

This image of James Watt's early steam engine shows a variety of inputs, outputs, and the connections between them. On the left side, the steam from heating water is the input to drive the big piston up, the output. That vertical motion, through the rod connecting the piston to the lever, is now a new input. The lever is a "rule" changing the direction of the motion connecting it to a wheel on the right side. This rule converts vertical motion to circular motion. The lights lines are shafts or belts to link the circular motion, yes, a link is a rule to drive some other outputs, one of which is the governor. The governor, that triangle shaped object with two balls attached in the middle of the diagram controls the speed of the engine spreading as it speeds up to close a valve to reduce the steam output or spreading to open it and to let more steam speed up the engine. This feedback is a rule modifying the input based on the output.



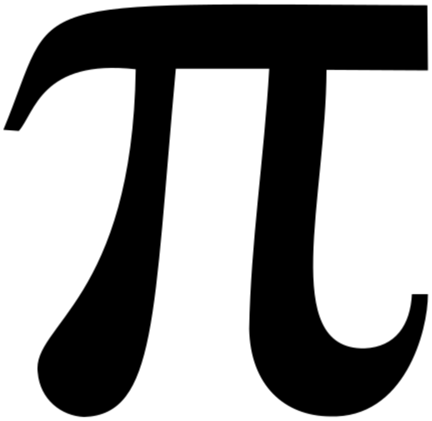
Patent Drawing of Watt Steam Engine



Governor

These are just a few of the functions that make up this function machine, converting heat to steam to drive a piston, to turn a wheel, to add more water, to… The function machines we build on spreadsheets work in the same way, sometimes just multiplying a quantity, sometimes changing one form of data to another, or sometimes using the output to control the input. Though we may think about different things today than people did 300 years ago, we still build our ideas in much the same ways. We still build models as collections of functions.

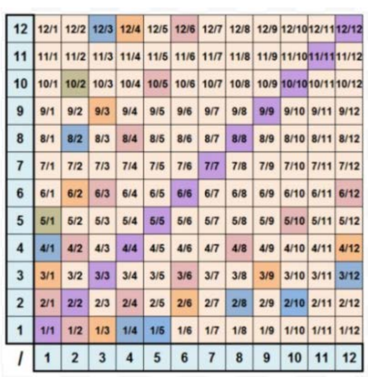
Functions Instead of Fractions



Today, division and fractions are the centerpieces of 4th and 5th grade Math Common Core. Ratios in their various forms, formats, and traditional problems are the focus of 6th and 7th grade. Fractions, taught first, serve as the foundation for working with ratios, and the paper algorithms for operating on fractions are used for ratios. The sequence made sense when the calculating technology used the paper algorithm, making fractions much easier to calculate than decimals. But when our calculating technology is digital reversing the order, moving ratio down to the 4th and 5th grades and fractions after it in 6th and 7th offers a number of significant advantages. It enables us to treat ratios as functions and fractions as ratios. It gives students the chance to be developmentally ready for the abstraction of a fraction. And it provides a much stronger platform for a problem-solving based curriculum.

What if Math -- Equivalent Fractions

It is not a strange idea to treat fractions as ratios. After all, fractions are rational (ratio) numbers. Fractions are numbers which is one of the reasons they are so difficult to understand. For unlike their sisters, the whole numbers, they are not unique or countable. How do we explain that ½ as the same as 2/4ths or 5/10ths or 1500/3000ths? How do we help students understand that 5/13 is bigger than 5/14? How are they to make sense of the operations when the product of two fractions is smaller than either one and the quotient is larger, just the opposite to what they learned with whole numbers? How do we get them to accept that to add or subtract fractions they have to first multiply? And without countability, how do they visualize the patterns of fractions, and make sense of their sequences.



12	12/1	12/2	12/3	12/4	12/5	12/6	12/7	12/8	12/9	12/10	12/11	12/12
11	11/1	11/2	11/3	11/4	11/5	11/6	11/7	11/8	11/9	11/10	11/11	11/12
10	10/1	10/2	10/3	10/4	10/5	10/6	10/7	10/8	10/9	10/10	10/11	10/12
9	9/1	9/2	9/3	9/4	9/5	9/6	9/7	9/8	9/9	9/10	9/11	9/12
8	8/1	8/2	8/3	8/4	8/5	8/6	8/7	8/8	8/9	8/10	8/11	8/12
7	7/1	7/2	7/3	7/4	7/5	7/6	7/7	7/8	7/9	7/10	7/11	7/12
6	6/1	6/2	6/3	6/4	6/5	6/6	6/7	6/8	6/9	6/10	6/11	6/12
5	5/1	5/2	5/3	5/4	5/5	5/6	5/7	5/8	5/9	5/10	5/11	5/12
4	4/1	4/2	4/3	4/4	4/5	4/6	4/7	4/8	4/9	4/10	4/11	4/12
3	3/1	3/2	3/3	3/4	3/5	3/6	3/7	3/8	3/9	3/10	3/11	3/12
2	2/1	2/2	2/3	2/4	2/5	2/6	2/7	2/8	2/9	2/10	2/11	2/12
1	1/1	1/2	1/3	1/4	1/5	1/6	1/7	1/8	1/9	1/10	1/11	1/12
/	1	2	3	4	5	6	7	8	9	10	11	12

What if Math -- Ratio & Proportion

Fractions are also difficult to work with in today's real-world problem solving. There are just not many interesting STEM problems or projects that require students to add/subtract/multiply/divide fractions, and fractions are rarely operated on in business computations because calculators and spreadsheets make decimals easy to compute. Fractions are most valuable today for solving headmath problems, which do not use the paper algorithms for computation.

NASA Planetary Data

	Mass (1024kg)	Diameter (km)	Density (kg/m3)	Gravity (m/s2)	Escape Velocity (km/s)	Rotation Period (hrs)
MERCURY	0.33	4879	5427	3.7	4.3	1407.6
VENUS	4.87	12,104	5243	8.9	10.4	-5832.5
EARTH	5.97	12,756	5514	9.8	11.2	23.9
MOON	0.073	3475	3340	1.6	2.4	655.7
MARS	0.642	6792	3933	3.7	5	24.6
JUPITER	1898	142,984	1326	23.1	59.5	9.9
SATURN	568	120,536	687	9	35.5	10.7
URANUS	86.8	51,118	1271	8.7	21.3	-17.2
NEPTUNE	102	49,528	1638	11	23.5	16.1
PLUTO	0.0146	2370	2095	0.7	1.3	-153.3

What if Math -- Solar System

Ratio, on the other hand, is essential for solving most of our everyday business and STEM problems that require quantitative reasoning. It is, therefore, very easy to incorporate ratio math in problem-based-learning activities that can involve rate, proportions, interest, probability, percentage, rate of change, and so much more to engage students in real problem-solving practice. As they develop their skills with ratio problems, students working with familiar elements, the counting numbers, and the already familiar connections between those elements– multiplication and division. At their basic level, ratios are concrete, the division of two numbers. Multiply one of those numbers by any quantity and to preserve the ratio multiply the other number by the same quantity.

When we deal with ratios as relationships between two numbers, we are treating them as functions. Since spreadsheets are function machines, they are a natural and obvious way to work with ratios. We can easily imagine students building equivalent ratios in paired columns, treating ratios as functions with two numbers an input and an output along with a rule (the connection) that divides the numerator by the denominator. We can imagine students using spreadsheet “Formats” to change the form of the ratio to decimal, fraction, batting average, rate, interest, percentage, money, measurement, conversion, etc. depending in most cases upon the denominator we choose. From the spreadsheets perspectives these are all similar ways of formatting the output of a function. They are all ratios. And from a student’s perspective they are all concrete representations built on quantities and concepts like place value that they have experience with.

Beginning with ratios in our math programs has significant advantages. Focusing on multiplicative operations gives students additional time to solidify their multiplication facts and provides the conceptual and the spreadsheet computational foundations for the real-world problem solving. Getting students to think about ratios as functions with inputs, outputs, and rules give them a consistent way to understand their various forms and formats, and familiarity with the vast array of ratio type problems we deal with in work and life. Perhaps most important, switching the order gives us the opportunity and tools to develop functions and functional thinking much earlier, to build the insight and the

Parameter Table	
$Numerator_0$	1
$Denominator_0$	1
Δx	1
x_0	1

Which of these fractions is bigger?

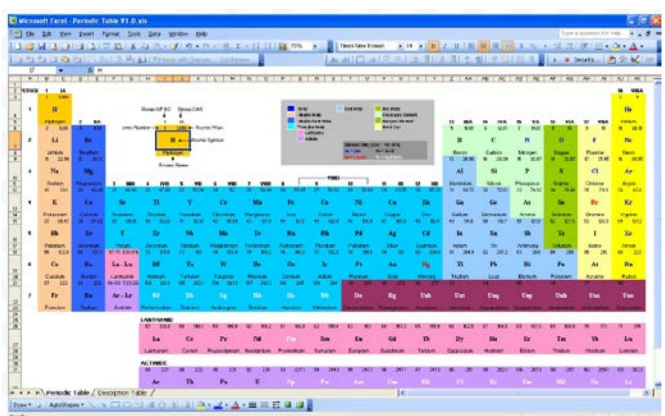
12	12	6	4	3	2 2/5	2	1 5/7	1 1/2	1 1/3	1 1/5	1 1/11	1
11	11	5 1/2	3 2/3	2 3/4	2 1/5	1 5/6	1 4/7	1 3/8	1 2/9	1 1/10	1	11/12
10	10	5	3 1/3	2 1/2	2	1 2/3	1 3/7	1 1/4	1 1/9	1	10/11	5/6
9	9	4 1/2	3	2 1/4	1 4/5	1 1/2	1 2/7	1 1/8	1	9/10	9/11	3/4
8	8	4	2 2/3	2	1 3/5	1 1/3	1 1/7	1	8/9	4/5	8/11	2/3
7	7	3 1/2	2 1/3	1 3/4	1 2/5	1 1/6	1	7/8	7/9	7/10	7/11	7/12
6	6	3	2	1 1/2	1 1/5	1	6/7	3/4	2/3	3/5	6/11	1/2
5	5	2 1/2	1 2/3	1 1/4	1	5/6	5/7	5/8	5/9	1/2	5/11	5/12
4	4	2	1 1/3	1	4/5	2/3	4/7	1/2	4/9	2/5	4/11	1/3
3	3	1 1/2	1	3/4	3/5	1/2	3/7	3/8	1/3	3/10	3/11	1/4
2	2	1	2/3	1/2	2/5	1/3	2/7	1/4	2/9	1/5	2/11	1/6
1	1	1/2	1/3	1/4	1/5	1/6	1/7	1/8	1/9	1/10	1/11	1/12
/	1	2	3	4	5	6	7	8	9	10	11	12

intuition so valuable for understanding ratio and proportion, the core real-world problem-solving requirements. And we are doing this with concrete numbers and common operations students already have experience with.

By thinking in ratio terms, we can approach the NAEP test problem–order these proper fractions from smallest to largest: (**2/7, 1/12, 5/9)** by building tables of ratios just as they built times tables enabling students to play with and grow denominator and numerators. Looking at the table in terms of slope from the origin, they can visualize and compare these fractions. And there are a wide variety of different *What if's* that we can ask and experiments that students can perform. For example, converting this table to decimals or to percents is a trivial problem in spreadsheets, and shows once again fractions as ratios.

Starting with ratios even opens the door to important ratios that are not rational numbers. For example, π is the ratio between the circumference of a circle and its diameter. Indeed, all the significant constants in our lives like **c**–the speed of light are ratios. By treating π as a ratio, a function, we help students visualize the circumference growing at π times the diameter. When we start with ratios instead of fractions, we not only open the door early to some wonderful and exciting mathematics for all students, math that can be concrete and fun, wide ranging, and valuable; we remove the barrier that blocks so many of our kids from achieving the success we and they dream of.

The STEAM Subjects Require Functional Thinking

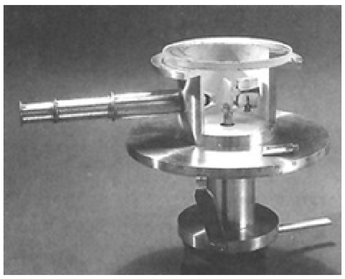


Periodic Table in Microsoft Excel

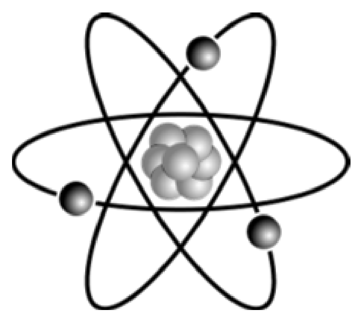
STEM, or STEAM as many now prefer, has become the acronym to describe the education and integration of the disciplines that are becoming central to the workforce. All except for one are obviously envisioned as laboratory courses. We cannot imagine learning to be a scientist without learning to experiment. We cannot imagine becoming a computer programmer without coding. We cannot imagine being an engineer without building something. And we cannot imagine being an artist or designer without ever working in a studio on a painting, a design, or with a camera. So why in this STEAM century do we find it not just acceptable but natural to spend 12 or more years in school learning math without ever stepping into a math laboratory, without experimenting, without creating, without asking “What if…”

Science

Galileo not only helped invent functional thinking, he applied it, inventing experimental science with his study of motion on inclined planes. Rolling balls down troughs he set at various angles, he measured the distance the ball traveled in units of time. By asking “What if…” of these inclined plane experiments, he diluted gravity and found its acceleration to be constant and independent of the weight of the objects he rolled. This first scientific experiment, asking “What if…” remains the archetype for all science.



We can see it at work in another canonical and important experiment. In 1911 Ernest Rutherford aimed a source of alpha particles (the nuclei of helium) toward a thin sheet of gold and from there to a scintillation screen that glowed when hit by these particles. Looking for the structure of atoms, Rutherford had his assistant Hans Geiger sit in a dark room staring for long hours through the microscope at that screen, counting dots of light and measuring the distance they were deflected from center. He found that most alpha particles went through the gold foil as if it were not there, with a few deviating slightly. When Geiger wanted to add Ernest Marsden to relieve himself of that boring task, Rutherford had a brainstorm, "Why not let him see whether any α-particles can be scattered through a large angle?" What if some of the particles bounced straight back?



Marsden found that some did! A very small number of alpha particles came back toward the source. From this experimental result Rutherford imagined a new model of the atom, the familiar atom we picture as the atomic energy logo. And using the deflection measures, he was able to calculate the size of the nucleus and of the atom itself. The atom is mostly empty space, much more so than is shown in this standard picture, with its nucleus is just 1/10,000th its size! It therefore looks nothing like this standard image most of us hold. Asking What if… in experiments leads us to build models of our world, models that in turn enable us to both explain and predict.

Experiments, the heart of science, are “What if…” questions. Scientists use experiments to build and test models searching for patterns. They think functionally, asking: “What if I change the input?” “What if the output were…? “What if I suggest a new rule governing that process?” “What if…” questions connect causes to effects and rules to models, laws, or theories. Einstein asked: “What if I rode on a light beam?” “What if I wanted to synchronize clocks with someone riding a train?” “What if I were in an elevator far out in space?” The “What if…” world of functional thinking is the world of the science laboratory both physical and mental. It is the world of experiment and theory.

Technology

I had the good fortune to practice coding when personal computing was in its infancy and microprocessors were much simpler. That simplicity was quite apparent in my Apple II microprocessor called 6502. Though BASIC was the language of choice for the Apple II, it was not fast enough for the things I wanted to program, so I turned to Assembly Language, the collection of three letter codes that represent the fundamental instructions built into the microprocessor. The 6502 had only about 50 such instructions, primitive by today's standards, they can be listed down a page and divided into essentially three groups and give us a sense of the simplicity of computers and programming:

ADC	add with carry
AND	and (with accumulator)
ASL	arithmetic shift left
BCC	branch on carry clear
BCS	branch on carry set
BEQ	branch on equal (zero set)
BIT	bit test
BMI	branch on minus (negative set)
BNE	branch on not equal (zero clear)
BPL	branch on plus (negative clear)
BRK	interrupt
BVC	branch on overflow clear
BVS	branch on overflow set
CLC	clear carry
CLD	clear decimal
CLI	clear interrupt disable
CLV	clear overflow
CMP	compare (with accumulator)
CPX	compare with X
CPY	compare with Y
DEC	decrement
DEX	decrement X
DEY	decrement Y
EOR	exclusive or (with accumulator)
INC	increment
INX	increment X
INY	increment Y
JMP	jump
JSR	jump subroutine
LDA	load accumulator
LDY	load X
LDY	load Y
LSR	logical shift right
NOP	no operation
ORA	or with accumulator
PHA	push accumulator
PHP	push processor status (SR)
PLA	pull accumulator
PLP	pull processor status (SR)
ROL	rotate left
ROR	rotate right
RTI	return from interrupt
RTS	return from subroutine
SBC	subtract with carry
SEC	set carry
SED	set decimal
SEI	set interrupt disable
STA	store accumulator
STX	store X
STY	store Y
TAX	transfer accumulator to X
TAY	transfer accumulator to Y
TSX	transfer stack pointer to X
TXA	transfer X to accumulator
TXS	transfer X to stack pointer
TYA	transfer Y to accumulator

- **Housekeeping** commands included things like: **LDA** and **STA**[65] to load and store data to special locations or to main memory or keep track of actions with counters like **INC** or **DEC** (increment or decrement).
- **Arithmetic** commands included things like **ADC** and **SBC** to add with carry or subtract with carry. Note there is no multiply or divide, for these operations were done by repeated addition or subtraction.
- **Logical** commands, the tools that make computers different from calculators, like **BCC**, **CMP**, or **JMP** to branch, compare, and jump to a subroutine build on the general "if…then" form, supported by **AND** or **ORA** for "and" and "or" logic.

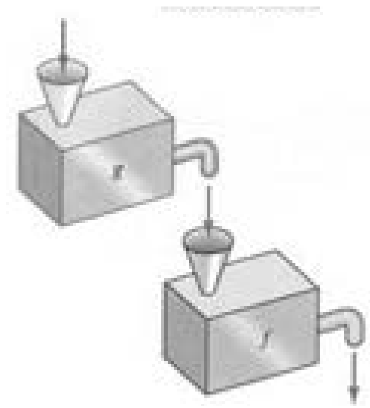
Technology and coding, at their very heart are functional "What if…," thinking. Spreadsheets and spreadsheet-based lessons remind us of the wonderful power coding can have on our minds; the deep desire to make the code work; to see, literally see, a model function; to feel the exhilaration of bringing imagination to life.

Coding, with its creative capability, (and spreadsheets are coding platforms), can engage every student in ways we just dream of today. For coding is functional thinking. As Ada Lovelace, credited with inventing programming, taught nearly two centuries ago:

> *"developping [sic] and tabulating any function whatever. . . the engine [is] the material expression of any indefinite function of any degree of generality and complexity".*

[65] Load Accumulator, Store Accumulator. The accumulator (an old name) is a built-in memory location where the microprocessor stores data awaiting the next instruction.

Engineering



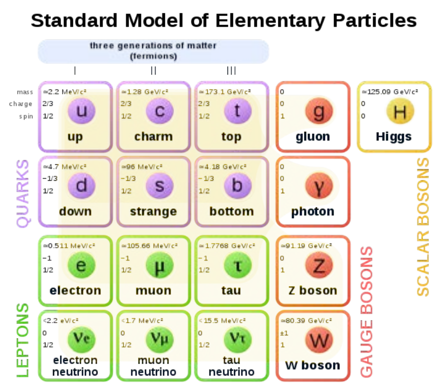
The term “mechanics” describes that arena of physics devoted to bodies in motion. Today, in physics, we talk about “classical mechanics”, “quantum mechanics”, “relativistic mechanics” and more. Mechanics is an old name that came from visualizing the universe as a great machine, a clockwork, with moving objects interacting. It is a symbol of the close association of engineering and the physical sciences. Much of physics and our physical intuition rests on this connection between machines and our concepts of nature. Einstein’s long stint as a patent clerk certainly built his incredible physical intuition, for learning to invent and describe new machines through concrete visualizations is a skill so often required to ground abstract conceptualizations.

Engineering is built on “What if…” functional thinking. Whether designing new bridges or inventing new phones, engineers spend their days building physical or conceptual models and asking “What if…” “What if this needs to support more weight?” “What if this part fails?” “What if I change this or make this smaller or make a car without a driver?” All machines have some kind of input and produce an output. They follow rules and engineers test and iterate those models. If you are applying for a patent, you will likely draw such a model as part of the application and show it in its various states as if it were a real machine, a fully functioning existing machine. This building of models whether in formulas, on spreadsheets, in drawings, or even in a functioning version is part of the patent process and an essential aspect of learning engineering.

The standard picture of function, even used with young students, has an input funnel, an output funnel, and a rule that resides in the box linking them. We ask them to imagine a machine inside that box performing some “mechanical” task like adding 3 to any input number, and we ask them, “What will the output be?” Or we play “guess my rule” when given both an input and an output we ask what rule will connect them. We even give them the output and the rule and ask what the input will have to be. This concrete image helps make even the most abstract function easier to understand, work with, and apply. This mechanical basis of functional thinking lends it a great power to build models concretely, like this one which shows the output of one function being the input of another function. Mathematics, even powerful mathematics, need not be just symbolic abstractions.

Art

What if… functional thinking is not limited to STEM disciplines. It is at the very heart of the arts and humanities as well. When we create a work of art we always ask: "What if… I change this color, transform this shape, use this key, explore this metaphor, play this faster, work in this style." When I use the term art, I envision the whole of the humanities. Poetry is just as much a "What if…" arena as science is. For poets teach us to make patterns and often through those patterns they give us the vocabulary and even the grammar to build other patterns. Perhaps the most famous example is the "quark", the invention of Murray Gell-Mann in 1964, who took this name for his new fundamental constituent of nature from James Joyce's Finnegans Wake, "Three quarks for Muster Mark!" Though George Zweig came up with the same concept independently, he called those new particles that made up protons and neutrons "aces." The symmetry embodied in Joyce's poetic line and in Gell-Mann's choice of words was emblematic of the symmetry in the theory, and Gell-Mann's name stuck.



We are all artists. I find that each of us has an art, at least one, that we love and enjoy. Mine is photography. I have loved taking pictures ever since I was a young boy. As soon as I could afford it, I bought a single lens reflex camera. I did a darkroom stint developing, printing, and enlarging black and white photos, some of which still compete in my home with the works of great professional photographers. But it was not until I started using digital photography that I was ultimately smitten. I began to take photos everywhere, enhance them in Picasa, print them in 8 ½x11 or 13x19, encase them in plastic for placemats or mount them on foam-core for walls.

I would play with each photo in Picasa, transforming it from an okay image, the input, to a work of art, the output, asking "What if…" I would change the contrast, saturate the color, crop the image, add more blue to the sky, or turn it orange. "What if…" I darkened the background or made it brighter, turned it to sepia, brought out one of the colors, or added shadows? The possibilities seemed to be endless, and this simple program enabled me to do so much "What if…" thinking that was sufficient for me. It was the technology that made the difference, it enabled me to easily and quickly experiment, to do and undo, to keep track of my rules, and to start all over when I got into trouble. The technology liberated the artist in me and enabled me to play, to experiment, to be creative and to share my work with others.

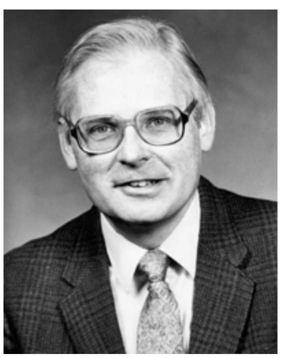
Enabling every student to be an artist as they learn mathematics would transform what for many is a dull feared subject to a loved one. It could make mathematics for every student a creative subject, and I believe there is great power in this human drive. As I found in photography, technology can liberate our imaginations and engage us all in creative patternmaking to make the learning of mathematics a creative experience.

Math

In 1988, a seminal article appeared in the prestigious journal *Science,* written by Lynn Arthur Steen the prominent American mathematician and educator, entitled "**The Science of Patterns**". Though his words now decorate our classrooms and our textbooks, and it is highly unlikely that any article in the history of this scientific journal had its title reprinted as often, they have had surprisingly little impact on math education.

In the article, Steen suggests that the digital age is causing us to redefine and indeed to reimagine the oldest of the disciplines. I believe it should also cause us to redefine and reimagine mathematics education.

> *The rapid growth of computing and applications has helped cross-fertilize the mathematical sciences, yielding an unprecedented abundance of new methods, theories, and models. Examples from statistical science, core mathematics, and applied mathematics illustrate these changes, which have both broadened and enriched the relation between mathematics and science. No longer just the study of number and space, mathematical science has become the science of patterns, with theory built on relations among patterns and on applications derived from the fit between pattern and observation.*[66]



Like any science, math should be viewed as a laboratory science in which students learn by experiment, by building and testing models, by looking for patterns in the data, and by using technology as tools for this exploration. His words suggest that we view learning and doing mathematics much like we would do and learn chemistry, biology, or physics. We find a problem, develop a model to be used to solve the problem, experiment with the model and iterate it to make it better predict the data, and ask, always ask, "What if…" as we extend the model and build a pattern of patterns. No longer a lone discipline segregated from the rest of STEAM; math in Steen's vision, becomes a natural and integrated part of a new whole where patterns and patternmaking are core ideas and skills.

[66] Lynn Arthur Steen, *Science,* 1988

The Near Perfect Tool

The office of the past

Leonardo's math is obsolete! Based on paper algorithms, it belongs in museums, not in classrooms. Now that we have made room in our math curriculum, what do we replace Leonardo's math with? Since Leonardo of Pisa based his mathematics curriculum on the needs of business in the 13th century should we not base our math curriculum and pedagogy on the needs of business in 21st century. Unlike the merchants and traders in medieval times, business today is broad and complex. It ranges from managing personal finances to running large corporations, from building a new app to developing major scientific research projects, from planning a classroom assignment to manufacturing the next generation of automobiles, from individual consulting to overseeing large government agencies. Yet, even when we think about business in the broadest sense of the word, we can find common ground in the kinds of problems faced and the quantitative means used to solve them.

The revolution in the mathematics of business that ended Leonardo's math reign began in 1979. The new personal computer provided the breakthrough technology for the invention of the spreadsheet by Bob Frankston and Dan Bricklin. Though at first, they thought of *VisiCalc* as a visual calculator; they designed it in the genre of computer programming language—as a function machine with inputs, outputs, and rules (formulas)—and soon came to recognize its "What if…" power. The spreadsheet enabled business to ask "What if…" as a natural matter of course, profoundly changing the way business operated. The math of business today is the building of models and not the calculation of solutions.

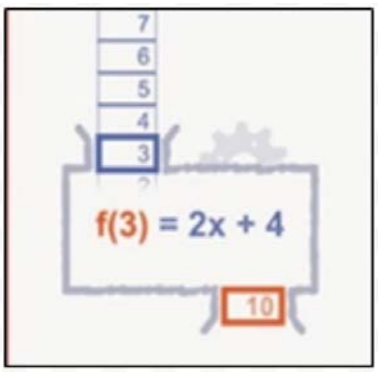
Bricklin and Frankston

The spreadsheet enabled individual entrepreneurs, executives, and managers to build their own models and to ask "What if…" as often as they wanted. It was a primary cause of the entrepreneurial revolution that began in the 1980's, built on spreadsheet generated business plans and venture capital expected returns. Soon after the spreadsheet's invention, Mitch Kapor, seeing the need for visualizing this new wealth of VisiCalc data and another way to represent functions, developed *Lotus 123,* integrating spreadsheets with graphs

and database tools, to build a complete function machine for the new business-oriented IBM PC's. The spreadsheet and the personal computer became a requirement for every business and a full realization of functional thinking. Excel, originally built for the revolutionary Macintosh in 1984, with its simplified interface, became and remains the standard quantitative data tool for all business and for most of science, technology, and engineering.

Today, the spreadsheet is the ubiquitous tool for business. It is a software offering from Microsoft, Apple, and Google. It is on every desktop, tablet, and smartphone. It is available on most every student computer, usually at no cost. Though a calculating device, list manager, and means to create pretty charts, the great power of the spreadsheet is in its capacity to engage students in "What if…" creative thinking.

Spreadsheets are Function Machines



Animated function machine by Ryan McQuade on whatifmath.org

Spreadsheets are function machines with inputs, outputs, and rules. An input cell can contain a number, word, or the address of another cell. An output cell has a rule. The spreadsheet knows it is a rule because it starts with an **=** sign[67]. Though spreadsheets do not use continuous variables or their symbols like **x,** they use cell addresses and tables like computer digital programs do for the same purpose.

Function is powerful. It not only quantifies cause and effect, it acts as an object. Like a number or a variable, we can perform operations on functions treating them as quantities to build complex functions out of simple ones. We can make the output of one function the input to another one or even to the input to itself (recursion). We can add, subtract, multiply, and divide functions, and even take a function of a function (composition). We can make functions with one variable or many. They are an incredibly flexible tool. Spreadsheets call the rule of a function, a formula, and collections of functions, a *model*; store them in *libraries* and use them as a coder would use *subroutines* with labels like **=SUM()**[68].

Spreadsheets Copy Functions Intelligently.

A function is not just the rule itself, it includes all its possible inputs, (its *domain*) and all of its possible outputs (its *range*).[69] Spreadsheets enable us to use copy and paste on functions and

[67] Unless you are entering or editing a rule, which spreadsheets usually call a formula or function they symbolize as $\mathbf{f_x}$, you don't see it because rules lie hidden inside the output cell.
[68] Sum a column of numbers where we simply fill in the cell addresses between the parentheses.
[69] Some versions of this picture use the word function to represent the rule inside the box. But a function includes not only the rule but the domain it applies to and the range it produces. So, I like this diagram in which the function represents the whole.

not just on their rules. Copying and pasting a rule thus normally includes the cells it expects to be its domain and range. This so-called *relative addressing* means that a rule copied from one cell to another or into a column or row would use the appropriate inputs. A rule can also be tied to a cell, a fixed *absolute address,* by starting the address with a $ sign. This cut and paste power with relative and absolute addressing lets us create complex and sophisticated models surprisingly easily and quickly. For example, if you wanted to create a monthly budget you can copy the column for the first month into all the rest. By doing so you are automatically copying its formulas like sum or average or percentage and with relative addressing these rules apply to the right data in each column. If some of the formulas refer to a single special cell, say one with your yearly salary, you can keep that reference in place by using its absolute address. This ease of model building and editing makes "What if…" questioning and experimentation easy and flexible.[70]

Since undo[71] is a simple idea available in almost every computer program, it is easy to overlook its great potential for enabling learning. Undo gives students the power to play and replay, to experiment and re-experiment, to make and remake. This is something many students need to learn to do and practice. Erasing paper is not easy on homework or tests, and many students worry about neatness, so few students are encouraged in today's classrooms to experiment, to play. Yet, we hear over and over from business that this willingness to try and fail, to take risks, to be wrong is a critical skill. Whether or not "fail fast, fail often" is good business practice, a willingness to take some risks is, and spreadsheets can help students learn this skill. **Undo** may therefore be one of our most powerful tools for building problem solving and creativity.

Spreadsheets Make Functions and "What if…" Thinking Concrete

Parameters

x_0	1
Δx	0

1															
	1														
	1	1													
	1	2	1												
	1	3	3	1											
	1	4	6	4	1										
	1	5	10	10	5	1									
	1	6	15	20	15	6	1								
	1	7	21	35	35	21	7	1							
	1	8	28	56	70	56	28	8	1						
	1	9	36	84	126	126	84	36	9	1					
	1	10	45	120	210	252	210	120	45	10	1				
	1	11	55	165	330	462	462	330	165	55	11	1			
	1	12	66	220	495	792	924	792	495	220	66	12	1		
	1	13	78	286	715	1287	1716	1716	1287	715	286	78	13	1	
	1	14	91	364	1001	2002	3003	3432	3003	2002	1001	364	91	14	1

For example, pick a cell on a blank spreadsheet, put in a rule that adds the cell above it to the one to its left. Copy that formula into a big area of the spreadsheet. Seed it by putting a 1 into the first cell. Go ahead and try it. This simplest of links blew me away when I first did it, and it continues to thrill me with the amazing links across mathematics from probability to binomial expansion, from number theory, to calculus, and more.

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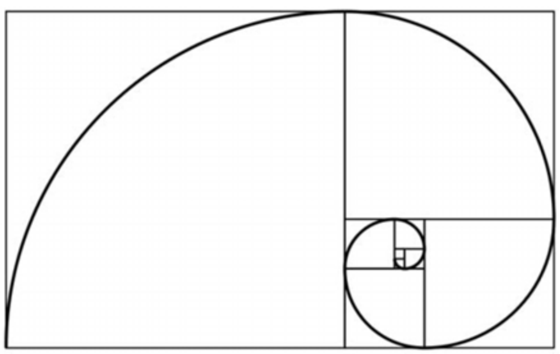
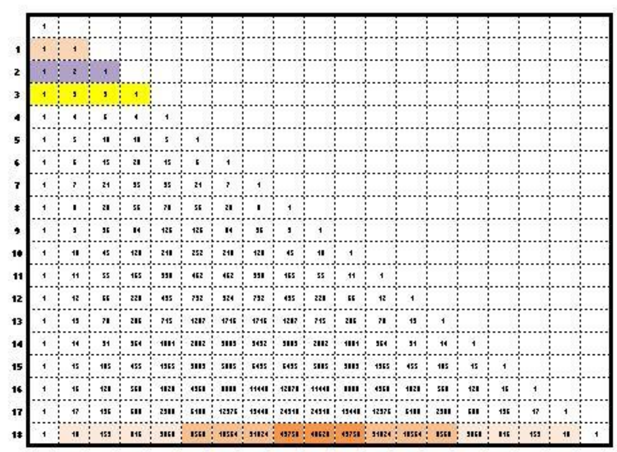
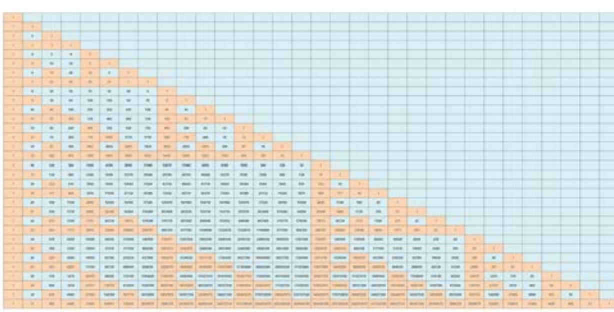
[70] This image comes from a fascinating video with Richard Feynman on mirrors and how they work.
[71] I am grateful to Steve Bayle for this insight.

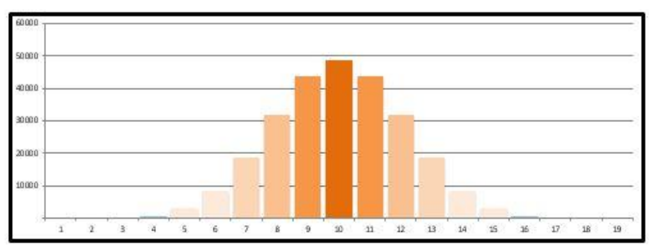
One of the most common complaints about math education today is that it is abstract. Focused on symbols and symbol manipulation, it lacks the visual patterns that tie it to the things students see and touch. Spreadsheets are uniquely positioned to connect the abstract to the concrete, the symbol to the visual, the problem to real data. Spreadsheets can automatically link formulas to tables, graphs, and images. These *linked representations* enable students with different learning modalities to understand and work with mathematical concepts.

Spreadsheets handle variables and functions discreetly as tables (columns or rows) of numbers. But they also link discrete (concrete) and continuous (abstract) functions in an intuitive way by letting students graph tables as discrete points and then convert the graph to a continuous line. And their natural linking through cell addressing lets us build amazing abstract patterns that we can explore concretely. This ability to link lets us graph one of those rows and get a Bell curve, add the numbers in a row and get a power of 2, follow a diagonal to get Fibonacci's sequence, or color the odd numbers to get a fractal pattern called Sierpinski. And this is just the beginning of the patterns found in one simple rule.

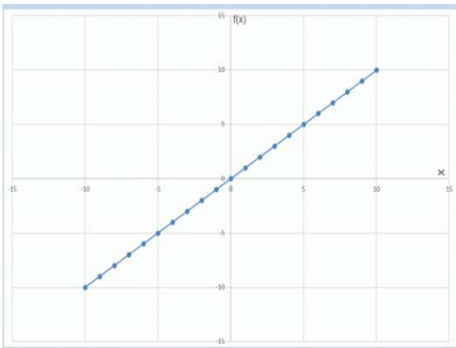






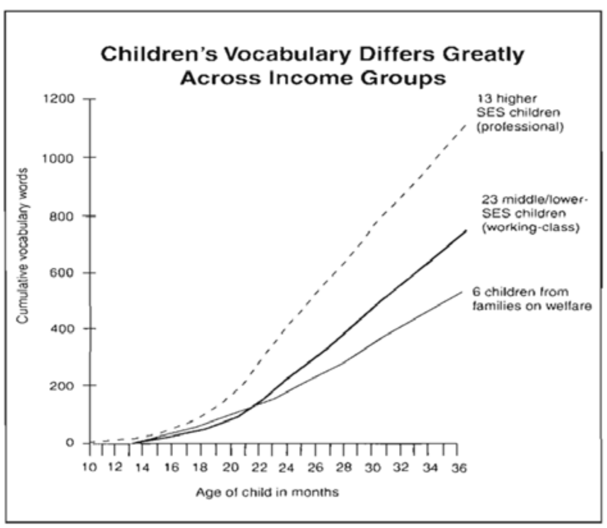


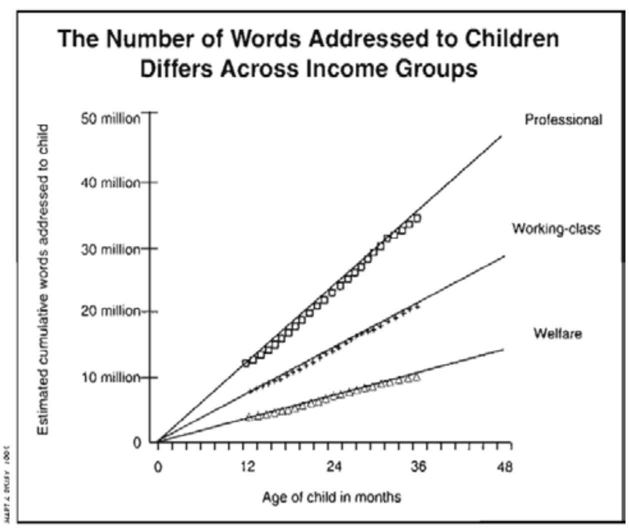
How the Failure to Think Functionally Distorted Reading Education



The graph of a linear function is a straight line

Graphs generally display functions, and they serve us in almost every area of our lives. They enable us to quantify cause and effect and ask the most human and predictive of questions, “What if…” A lack of understanding of functions and their visualizations as graphs can lead even the finest researchers and teachers to reach faulty conclusions. Today, when science and scientific evidence pervades educational thinking, this lack of understanding of function and experience in thinking functionally can lead to distortions of even the most important research findings and just plain wrong applications of “scientific” evidence. This, I believe, has happened to the research on reading that today drives and distorts preschool and ELA educational policy. It is a case study in the need for all of us to learn to think functionally.





The Word Gap

The Hart Risley study, originally published in 1995[72] and arguably the most important study in the development of reading ever published, was the culmination of 40 years of research by Betty Hart and Todd Risley. They put tape recorders in the homes of 42 families over a three-year period and transcribed the language interactions between parents and children. They found that socioeconomic status had a profound effect on the number of words children heard

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72 “The Early Catastrophe: The 30 Million Word Gap by Age 3”, Hart & Risley, American Educator 2003

and used. When they later followed some of those children through elementary school, they concluded that this word count was a major predictor of reading attainment and thus of school success.

Though the headline from their work “The 30 Million Word Gap,” captured most of the public’s attention, it was their graphs that have had the most profound and lasting effect on schools. The number of words addressed to children (the graph on the left) shows a linear increase with age. The children’s vocabulary (the functional graph on the right) shows the number of words they know and use. It starts out slowly and then around age two explodes when most children begin to first talk in two-word sentences and then suddenly burst into grammatically correct full sentences. At that point, their “use” vocabulary grows rapidly and linearly (in a straight line).

From these graphs two conclusions were drawn. First: that vocabulary growth was linear starting around age 2. Second: that once its rate, its slope, had been set by about age 2, it would require, as a child gets older, increasingly massive intervention to change that rate of vocabulary growth for the lines continue to diverge. The reading field believed that vocabulary size was, for most children, the key determining factor in reading and thus in school success. It pegged a 5,000-word minimum for learning to read fluently in 1st grade and a 15,000-word minimum to develop the quality of reading necessary for high school success. These linear functions, extrapolations of the Hart Risley graphs, suggested that students needed to add about 1,000 new words a year to their vocabulary to be successful in K-12 and college ready. This suggested the mantra, “Learn to read, read to learn” which has defined the ELA curriculum for the past two decades – focus K-3 education on learning to read and 4-12 on reading more and more advanced texts to build that vocabulary, i.e. read to learn. And they clearly identified, as the fundamental educational problem, that children who grow up in physical poverty will also likely grow up in vocabulary poverty.

This vision of vocabulary growth as a linear function produced a powerful impetus for vocabulary building in preschool and parenting education emphasizing talking to babies[73]. It further led to a massive focus on reading in the primary grades in both general and remedial classrooms with reading fluently by 3rd grade as the primary educational goal for all. For as children grow older the “achievement gap,” the divergence of those linear functions, increases. Thus, more words had to be learned more rapidly just to catch up to the faster growth rate line. All attention became focused on helping students close the gap, to make up for the deficit.

As with most single focus requirements, the unintended consequence of these graphs has made elementary school reading, the focus of school activity and insidiously tells students that if they do not get on the right track, they will be relegated to a life of poverty and low status. These linear graphs strongly suggest that if you do not get on the right slope when you are young, or, do not get boosted with a great deal of effort early enough, you will never make it, never build the vocabulary and thus the reading skills you need to get through high school, let alone to get

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[73] Betty Bardige, *Talk to Me Baby! How You Can Support Young Children’s Language Development,* Brookes Publishing Company, 2016

into college or enjoy a good career. This harsh and extreme vision most certainly did not represent the views of the fine researchers in reading education, but often their much more nuanced conclusions were lost in the image and understanding of a linear function.

These now widespread catastrophic alternatives and school obsessions require us to be very sure we understand this vocabulary development function. Linear functions are so pervasive and so deeply human that we have a hard time questioning them in any area of our lives. But for our children, as far as reading is concerned, these straight lines become defining lifelines. When you did not hear enough words as a baby, when you did not go to a good preschool, when you did not get the proper intervention in the primary grades, you will not be a good reader and very likely not be successful unless, of course, you are a rock or a basketball star.

Yet, let's step back for a minute and look at this linear function. Does it make sense that vocabulary growth remains linear after age 2? Are we adults still learning a thousand words a year? When did we stop gaining vocabulary at that incredible rate? When does the curve bend and if the function is not linear what does it look like? Has the reading field fixated on the wrong function?

As an adult, it seems to me highly unlikely that I am adding 3 new words a day to my vocabulary.[74] I would be very lucky to add 3 new words a week, 150 new words a year. At some point the line has to bend and bend radically. A function that has a linear component but that bends radically would look like a traditional learning curve, sometimes called the **S**-curve. Like the original Hart Risley vocabulary graph, a learning curve would start up slowly, at a shallow slope, increase slope radically at some point when it enters a high rate-of-growth and then bend again to slow down to a low rate of nearly linear progress. Learning curves are the ones we commonly associate with mechanical skills like riding a bike, driving a car, sewing and knitting, fixing things, or growing things. We begin learning slowly as we build understanding or muscle responses. When we reach a point where we start to "get it" we learn rapidly and improve very quickly. Then, as we approach mastery, our learning rate drops considerably and continues at that much slower pace perhaps even goes to **0** slope.

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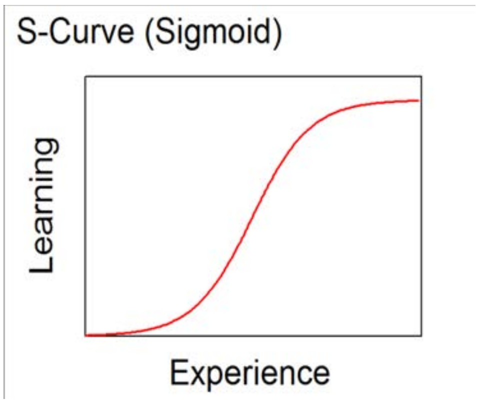
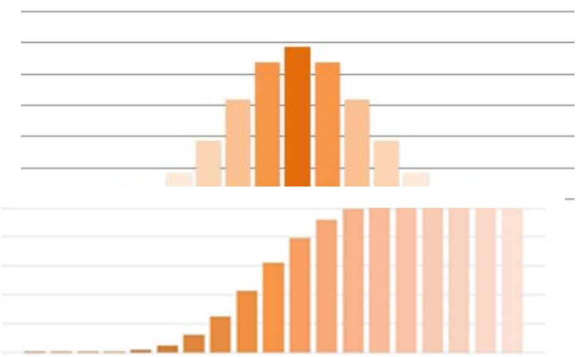
[74] New words, I suggest, should not and do not include proper names. Proper names do not need a definition, they are not concepts which must be built and understood. They are concrete references to objects. Watching a football game, we may be introduced to as many as 100 new names in a couple of hours. Proper names are stored in a different way in the brain, and we often skip over them without decoding them when we read. And as we grow older, we likely find it harder to remember and connect those proper nouns.

The Learning Curve

What if… this new function, this learning curve[75] function applied to reading? What if we began building our vocabulary slowly and then suddenly rapidly increase the speed at which we learned new words and having gained enough new words to read fluently, we slowed down again as we had mastered reading and knew most of the common reading words in the language? The bottom of the Hart Risley vocabulary curve does look like the bottom of a learning curve. The center of a learning curve looks linear like most of the Hart Risley graphs, and if we are right in assuming that vocabulary growth does indeed slow down significantly at some point then the top also could well fit the top of the learning curve. Without more empirical data we cannot be sure that reading/vocabulary building is a true learning function, but even without more data we can certainly conjecture that the learning curve is a much more likely representation of vocabulary building than a linear function.



The learning curve is not arbitrary. It is not just a nice drawing we make to picture and represent what we think is the acquisition of some mechanical and perhaps cognitive skills. The learning curve is an important function, the integral of the Normal distribution. The Normal distribution or "Bell" curve is perhaps the most important of all statistics. It is the probability distribution of random events. It represents an extraordinary variety of our experiences from measures of intelligence and learning to measures of sizes and shapes. This function is familiar to us all.



An integral is an area, in this case the area under the Normal distribution. You can find the integral by thinking of the normal distribution as a bar chart, with bars increasing in height from left to right until they reach a maximum at the mean and then decreasing in height until they get very small along the right tail. The area under the graph at any point is just the sum of those bars. You can picture that sum staying very small in the long tail on the left side, then raising quickly as the bell curve approaches the mean. At the mean the integral hits a "point of inflection," a point where the curve starts to bend in a different direction. The learning curve is now beginning to level off, slowly at first and then rapidly, never going to zero growth but improving very little as we add up

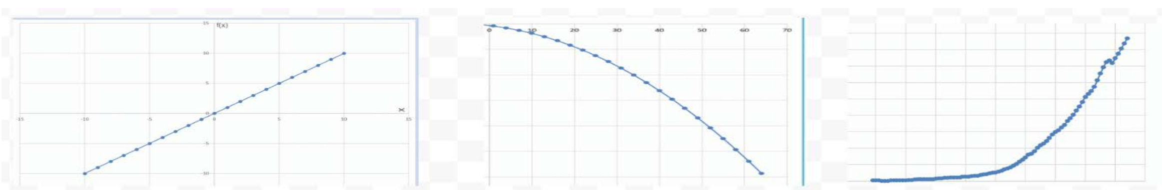
[75] There are no standard definitions of a learning curve or even the **S**. I have chosen the most common form, in the shape of a slanted **S**. This is the traditional view of process learning.

the bars in the very long right tail of the normal curve. The learning curve is thus very important and deeply connected function.

Applying the learning curve to reading suggests that building vocabulary is a process that starts slowly and builds more and more rapidly until it reaches a peak, the mean of the Normal distribution and the point of inflection[76] of the learning curve, and then the rate at which vocabulary growth slows until it tails off over a long lifetime as mastery in reading and speaking is reached. When do these points occur? We can make some good guesses and suggest some valuable research studies.

Implications for our Schools

If this is the right function, it has profound implications for education. It suggests that until a student reaches her "inflation point", slow vocabulary growth may not be a serious problem because the inflation point where the slope begins to increase rapidly will accelerate vocabulary growth and with-it reading fluency. It suggests that whenever a student has the rapid growth spurt, he will be able to get to the word count and fluency necessary to succeed in high school and college. It suggests that we may well be putting too much emphasis and pressure on early vocabulary building and on early reading success, for a student will be able to catch up when ready. If the **S** function, rather than a linear function, is the right interpretation of the Hart Risley data, then we may no longer have to focus on intervention to move students to the "right" rate of growth, but rather be patient and encouraging, wait for readiness at most any age to climb the accelerated section[77]of the **S** curve to reach the vocabulary needed to succeed.



Linear Function　Quadratic Function (parabola)　Exponential Function

Children brought into a new country with a very different language generally have little difficulty mastering the new language and building a huge vocabulary in the new language if they have the concepts from their native language. They almost instantly enter the inflation[78] portion of the learning curve, and by my reckoning are fluent readers in their new language within a year. It is not a surprise that reading scientists, reading educators, and teachers thought vocabulary development a linear function.

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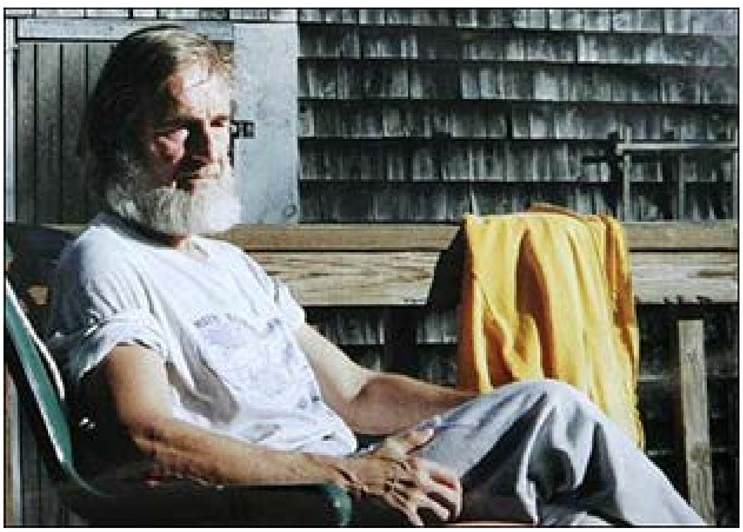
[76] In mathematics, point of inflection has a very specific meaning. It is where the curve bends in a different direction, as Wikipedia defines it: "the curve changes from being concave to convex, or vice versa."
[77] Since the learning curve is the integral or sum of the normal curve, then the derivative or rate of change of the learning curve will be the normal curve, then the peak acceleration (rate of change) of the learning curve is the center of the normal curve. One more way functions help us picture nature.
[78] I am using inflation here like it is used in astrophysics, as that portion of the Big Bang in which the universe grows dramatically faster for a short time.

Most of our algebra education is about linear functions. And most of the interpretations we see and hear about are linear functions. We expect them, we find them easy to use, and we use them as our standard relationship pattern. But not all functions in our world are linear. Gravity near the Earth makes projectile motion quadratic. And compound interest makes loan costs and even government spending exponential.

If learning new words, learning to read, like most physical learning is an **S** function, we need to rethink how we teach our children to read. And perhaps even more importantly to rethink the mathematics we must teach our children, for if our education scientists do not have a fundamental grounding in functions and functional thinking, then they will be prone to developing faulty patterns to explain learning.

"Algebra before Acne"



As I reread the Common Core Standards for the umpteenth time, I am struck by its introduction of variables in grade 6. Jim, I couldn't help but think of you, dear friend, and your wonderful command, "We must teach algebra before acne."

Kaput[79] envisioned algebra and algebraic reasoning as fundamental mathematical ideas that should be taught from the very early grades. He believed the great abstractions, the patterns which make mathematics so powerful and so beautiful could and should be taught from the very beginning. He helped write the original NCTM[80] Standards seeking to redefine mathematics education in the 1990's. He would not have been happy to see these new Standards. The Common Core had taken traditional abstractions and just moved them down a grade or two without providing ways to enable all students to learn and understand them. He would have been frustrated with the continued siloing of variables as the concept that dominates the math curriculum *after* arithmetic.

Through much of the 20th century algebra was the high school gateway course. Freshmen were split between those that entered the algebra sequence and were college bound, and those that stayed in the arithmetic program and either got a high school diploma or dropped out. Over the past quarter century algebra has started appearing in 8th grade and now as a 7th grade course to

[79] Dr. James Kaput, UMass Dartmouth Professor of Mathematics and brilliant visionary.
[80] National Council of Teachers of Mathematics

prepare some students for AP Calculus in their junior year.[81] Algebra remains today, the gateway course, but now for middle school. If you are good in algebra you are pretty much automatically college bound, quality college material, and candidate for a STEM career. And as it increases in importance, it is now demanded of every student and has become an integral part of the 7th and 8th grade curriculum in the Common Core forcing tracking even earlier.

From the perspective of Piaget, algebra and variables required students to be formal operational. From the perspective of students, they now ask the painful question, "What is **x**?" From the perspective of teachers, we wish there was a simple answer. From the perspective of math historians, we make the excuse that it took 800 years for the "unknown" of al Khwarizmi to become the variable of Descartes. So, we wonder why we should expect a topic that 60 years ago top students learned in high school can now be taught to all students in middle school. that Kaput could possibly expect all students to learn in elementary grades. We might think Kaput a dreamer who would oversimplify, even more, this abstract idea to present it even earlier. But we would be wrong. We would miss his genius. And we would miss a great opportunity to give all young students interesting problems to solve.

We are so wedded, in the standard math curriculum, to dealing and thinking of algebra and its variables as continuous quantities that we do not recognize the concrete power and utter simplicity of dealing with variables as discrete quantities. Students have no problem with this, after all arithmetic is all about discrete quantity. I did not recognize this until a decade after Jim's untimely death, when I started working "spreadsheet math." Spreadsheets are a natural medium for dealing with discrete quantities. Variables are represented by tables of values, a row or a column of discrete numbers. To operate on a variable is to operate on each number in turn. Functions are discrete as well, and generally link an input column to an output column. Indeed, in the application of math today in both STEM and business, spreadsheets are the primary quantitative vehicle, and discrete variables are the standard quantities. Spreadsheets are digital tools and as such are built to handle discrete variables and functions.

	Count On Table											
Input	1	2	3	4	5	6	7	8	9	10	11	12
Output	3	4	5	6	7	8	9	10	11	12	13	14

If we ask students to build a table of values from 1 to 10 on a spreadsheet, and label that row **x**, then "What is **x**?" It is simply the name of that column! It is a variable because it can take on each of those different values, any of those values, all those values. And if we ask them to make a second row to add 2 to the variable **x**, they will have no difficulty creating a "function of **x**" labeling the second row **f(x)**, a function machine that adds 2 to every value of **x**. First graders can do this. We can teach algebra from the very beginning if we use discrete variables and spreadsheets. Kaput was right, we can, and we should teach algebra before acne. And perhaps those, like myself, who believed algebra required formal operations were wrong. Technology give us the tools.

[81] Because the AP Calculus test has become a prime criteria for college admissions, a recognition of its theoretical difficulty.

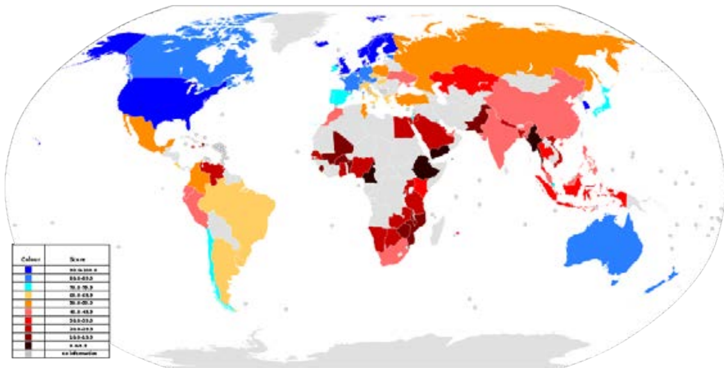
5. Learning Math as an Experimental Science

"The 'Melancholia' is one of the three most famous copper engravings of the great Nuremberg painter, draftsman, and engraver Albrecht Durer ((1471-1528). The authors of this book hope, therefore, that they may be permitted to take the brooding figure, sitting amidst a litter of mechanical tools, scientific instruments, and mathematical symbols, as the embodiment of the spirit of physical science.

Physics is an experimental science, as suggested by the tools—hammer, plane, saw—at the foot of the winged figure. Thus, by means of a few simple experiments with string, balls, and wax, GALILEO did more to discover the actual facts of math than had centuries of mere observation."[82]

[82] Frontispiece and text to *Mechanics Molecular Physics Heat and Sound* by Robert Millikan Nobel Prize Physicist, Roller and Watson, Ginn & Co. 1937

The Digital Age



We often confuse and confound the Internet and the Web. The Internet was a group invention that came out of a desire by researchers at U.S. government labs and their university partners to easily communicate with each other and to do so in a fashion that would protect them in the case of war. It is a flexible extensible communication network. As a technology it was brilliantly conceived, but as a concept it was not new.

The Web, on the other hand, was the work of one man, and a brilliant new conception that we do not even find imagined in our best science fiction. It was not even suggested in either Star Trek or Star Wars. It was imagined, named, and originally programmed by Tim Berners-Lee. At its heart, it was an instantiation of hyperlinking an idea that had been floating around since the 1940's.

I first came to know hyperlinking when Apple brought out an extraordinary system, they called HyperCard originally envisioned and programmed by Bill Atkinson. HyperCard was imagined as a stack of 3x5 notecards that could contain text, images, video, and more in which any word could be linked to another card as a means of elaborating on it. The hyperlinks were limited to other cards in the stack. It was a brilliant invention that I saw used in a variety of educational ways like solving math problems and teaching physics concepts. It was in those days limited to stacks of virtual cards that were saved and shared on floppy disks. The hyperlinking was brilliant but not extensible, and after a relatively short explosion, this promising technology faded away.

Tim Berners-Lee was not looking for a brilliant technology. He was searching for a solution to a nagging problem. Working as a young physicist at the huge CERN labs in Geneva Switzerland with 10,000 people in their phone directory and 5,000 resident there at any one time, he took on the problem of the proliferation of scientific papers and the need to communicate whether on campus or back home at their own institution. The new Internet was making the sharing of papers much easier, but that very sharing was making the library function overwhelming and unmanageable. In 1989, he proposed the Web[83]:

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[83] Web Index – https://en.wikipedia.org/wiki/Web_index

…the driving force I had in mind was communication through shared knowledge, and the driving "market" for it was collaboration among people at work and at home.[84]

He continued:

By building a hypertext Web, a group of people of whatever size could easily express themselves, quickly acquire and convey knowledge, overcome misunderstandings, and reduce duplication of effort. This would give people in a group a new power to build something together.

A web of knowledge linked through hypertext would contain a snapshot of their shared understanding.

The intention was that the Web be used as a personal information system, and a group tool on all scales, from the team of two creating a flyer for the local elementary school play to the world population deciding ecological issues.[85]

Less than 30 years later, his dream is a reality, a reality of great power. It is this power of the Web, of hypertext, of links, already a natural part of the lives of our kids, that can enable the vast majority of them to thrive in this new age. It is this tool, this concept, that we want our kids to learn to use, to create their own web of knowledge, and to creatively solve the problems they face. This is why I believe that every student who leaves high school should have created and used a website as part of their portfolio to understand how to think and present with hyperlinks. And to go even further, their schools, their classrooms, and their learning should be "open-web". Links are the digital age's essential communication and learning tools.

Model Building

As a youngster, I loved nothing more than model building. My father would take my brother and me on a special outing once or twice a year to The Hobby Shop owned by his cousins Rosie and Louie. In that dark dusty basement store piled high with boxes of models in less than perfect stacks, I thought I was in heaven. I would diligently search for interesting wooden model kits to build, while my younger brother Steve sifted through, with less enthusiasm, the plastic ones. We would gather our small collections, Louie would throw in a tube of the right glue into each box, and I would walk out of that store having been given the most precious gift. I loved wood models, model ships, model planes, model cars, and disdained the plastic ones that in my view took no special talent to assemble. By the time I was just 7 years-old, I was already taking a

[84] Tim Berners-Lee, *Weaving the Web*, 2000, p 162
[85] Ibid p.162

double-edged blade from my father’s Gillette dispenser, covered one side with a piece of masking tape to fashion a razor knife for cutting out the model pieces from thin planks of soft balsa wood.

I find in that hobby the model for my vision of digital learning. I loved the wooden model because even though it had a complete set of directions, I could control and change anything I wanted to. It was something I built, not just assembled. It was flexible and I could explore it and even experiment with it. It taught me care and grit, for it was easy to make a mistake and it took concentration and persistence to cut out all those wing spars carefully so as not to damage even those very small and fragile pieces. And though there were instructions in each box, they demanded that I think hard and visualize the connections between the pieces before applying the glue. I had to start by checking out all the pieces in the box, visualizing the final product from the picture on the box, then build each component in turn, the wings, the fuselage, the tail, before assembling the whole.

This is the model of the process I picture students going through as they work on digital age lessons. Choose things to build or to solve from a vast collection of interesting ones. Imagine the result and visualize the process. Build the model piece by piece, then assemble the final product, adjust it, paint it, and massage it to perfection. Then fly it around their room and hang it on threads from the ceiling for all to see. It is bottom up; making each section in turn and then putting the whole together. It is built using adult tools and materials that unlike preformed plastic parts can be changed. It is a creative process, an experiment, really an adventure. This what we have been working on in *What if Math* Labs. I believe this is a powerful model for lessons in the digital age. In the schools of tomorrow, we need to be giving our students and teachers “wooden kits” to build their models, from the bottom up.

“What if Math” – A Test Kitchen

Chefs tell us that cooking and baking are profoundly different activities. They require very different ways of thinking and working. Though for both, the goal of a delicious product is the same, the way they reach that product differs dramatically. Good bakers take well-defined recipes, carefully select ingredients, and rigidly followed procedures. Good cooks do not have to wait for their product to be finished to taste it. They can play, change recipes as they cook, add new ingredients anytime, and invent new processes midstream. Like most generalizations, this one certainly has its limits, but these two distinct ways of preparing food provide us with a useful dichotomy that we can connect to digital learning. For while both can certainly be creative and experimental, a baker must prepare the experiment completely and not deviate once started[86]. A cook can play and change and try something new during the process.

So too with curriculum and learning. Some curriculum, particularly the stuff of textbooks, is baker type learning. It lays out a formal plan and even when it tries to be discovery-based, it

[86] Except perhaps for decorations.

makes the student carefully follow a defined sequence to a previously determined conclusion. Because textbooks have been the curriculum source for nearly two centuries, technology-based curriculum and instruction continues to follow the baker model with a fixed recipe, scaffolded to lead the student through selected ingredients and procedures to a preordained product. Lesson plans are therefore the recipes that teachers are given to follow. Materials, links, and selected Webpages are the ingredients, the limited selected ingredients teachers and students should use. Goals are well-defined and driven by summative assessments like the outcomes of a baking competition.

Imagine instead, a cook's vision of a digital age curriculum and instruction. Imagine giving students and teachers powerful tools like spreadsheets or WordPress, an Internet rich in content and ideas, and a wealth of interesting problems to choose from to enable students to experiment as they are cooking, to play, to iterate, to ask "What if…" and to follow different paths as they add ingredients, taste and sample all the way to completion.

There are certainly times when learning should follow the baker's model. It can, in many training situations, be the fastest way to mastery. But for the most part, schools should follow the cook's model, for we want our kids to learn how to solve novel problems, to learn how to learn, to think of their classroom as a laboratory. Thinking as cooks we will begin to iterate the models we tentatively put together and build a coherent vision of the future of education.

I start with math. Not only is it the school subject I know the best, having been involved in math education directly and indirectly for 50 years, but It is the centerpost in our schools, often defining the placement of students. It may well be the most important subject of the digital age since it is crucial to STEM and shown by research to correlate highly with success even in reading.[87] But perhaps the most important reason I chose to start in math education is the pain I have witnessed over the years in the faces of students who did not feel smart because of math, and who therefore struggled to get the education and degree they wanted. During the past 5 years, I have worked with a small group of likeminded, dedicated, visionary friends asking:

What if we, like Leonardo, were to reinvent math education for our age, our digital age?

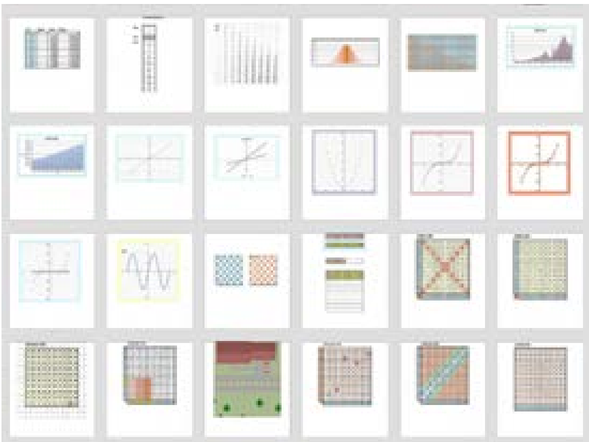
We have been building a math focused STEAM program on the foundational elements laid out here to help explore and define the future of education. *What if Math* is designed to:

1. Prepare our kids to meet the needs of 21st century work and life.
2. Use spreadsheets, the ubiquitous business technology of the digital age, for calculating and model-building.
3. Focus on functions and functional thinking for problem solving.
4. Be a laboratory course because mathematics is a "science".
5. Foster the 4Cs (Creativity, Critical Thinking, Collaboration, Communication).
6. First and foremost be a creative force making learning a creative experience.

[87] National Governors Association paper 2014

7. Enable widespread contributions to its content to…
    a. Give students real choices of problems, topics, and contexts to naturally engage them.
    b. Develop both headmath and handmath skills
    c. Ensure that all students are prepared for the future by practicing solving real problems.
    d. Enable students to learn on their own and in collaboration with other students.

Functional Thinking



While we suggest a rich and varied landscape of content for students to have choice, we also believe that their problem-solving process must be standardized. If students are to be able to work on lessons on their own or in small groups they cannot be burdened by a myriad of instructions or confused by varieties of procedures. For lessons to be transparent, students have to quickly and easily find a path through them. For problem solving practice to be valuable it must have a standard procedure, a common model, that gives students the latitude as well as the opportunity to be creative. We developed a standard problem-solving methodology to give students the ability and the common language to imagine, collaborate, and solve a very wide range of STEM and business problems. We call the process, **functional thinking** and base it on design thinking[88]. Here is an example.

We call this legacy motion problem, George and Martha. Though it does not represent digital age problem solving, it did lead us to understand and build the functional thinking process. We pose the problem in the traditional way:

> **George is in New York and Martha is in Washington. They leave at the same time and follow the same road to meet each other on the way. George has a fast horse and averages 16 miles/hr. Martha has a slow carriage and averages 7 miles/hr. How far will George have gone when they meet?**

Rate or motion problems are often found on math tests and in real world instances. Though this is a more traditional problem, if you can solve it you can solve them all if you think about them functionally.

Parameter Tables

distance	miles
New York	
Washington	
rate	miles/hr
George	
Martha	
time	hours (hr)
t_0	
Δt	

1. **Visualize** George and Martha's trip on a map. **Use the Web to find the road they can take and the distance they will travel.** What do you expect for an answer?

2. **Organize** data by building and filling in a parameter table. Build your input table using a rule with a starting value $\mathbf{t_0}$ and increment value $\mathbf{\Delta t}$.

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[88] Stanford School of Design

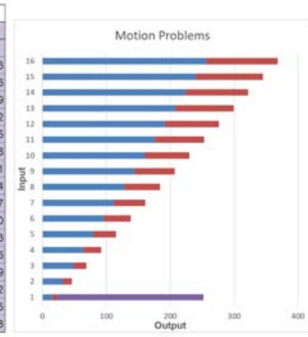
3. **Build a model** by filling in the function tables to find the distance George and Martha go every hour.? Build a 3rd table that shows where and when will they meet. Add a graph that will help you visualize the solution. What is the solution if Martha is not moving? What if George is moving 3 times as fast as Martha?

Parameter Tables

distance	miles
New York	0
Washington	229
rate	miles/hr
George	16
Martha	7
time	hours (hr)
t_0	1
Δt	1

Function Tables

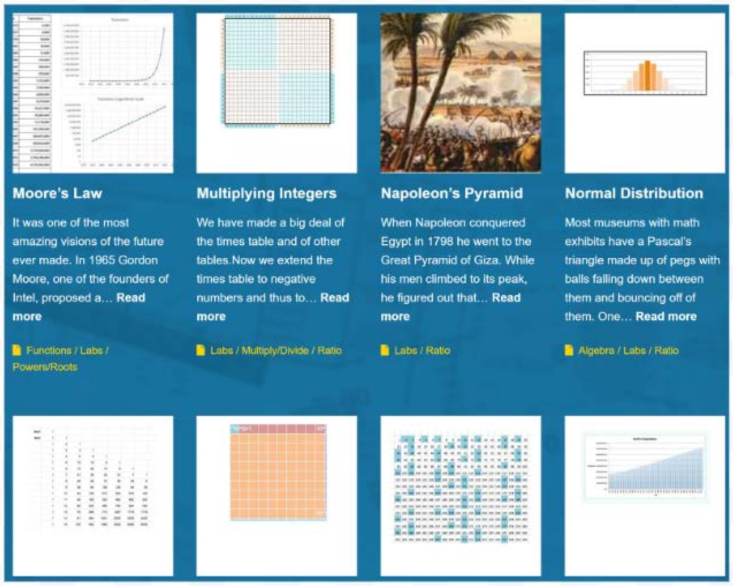
George		Martha		Total Distance	
t	g(t)	t	m(t)	t	d(t)
hours	miles	hours	miles	hours	miles
1	16	1	7	1	23
2	32	2	14	2	46
3	48	3	21	3	69
4	64	4	28	4	92
5	80	5	35	5	115
6	96	6	42	6	138
7	112	7	49	7	161
8	128	8	56	8	184
9	144	9	63	9	207
10	160	10	70	10	230
11	176	11	77	11	253
12	192	12	84	12	276
13	208	13	91	13	299
14	224	14	98	14	322
15	240	15	105	15	345
16	256	16	112	16	368



4. **Iterate** your model to increase its accuracy, let's say to one decimal place. How would you change **t_0** and **Δt** to find an exact answer?

5. **What if...** George left 2 hours later than Martha, where will they meet? How would you change your model to solve this problem? Invent another motion problem and use your model to solve it.

Explorations



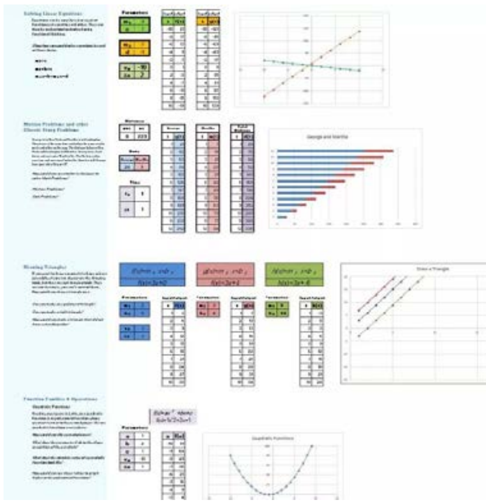
What if Math Explore menu

We call our lessons Labs so that both students and teachers see them as experiments and explorations using the spreadsheet as their math laboratory. At *What if Math* (whatifmath.org) students can choose from a growing number of Labs that run the gambit of content across mathematics, grade levels, and STEAM subjects. We designed the Explore pages to highlight and organize those choices. We treat the Labs like blog posts with an image and accompanying descriptive text.

Like music and app sites, Labs are arranged in a variety of ways: alphabetically to display the range of choices, by topic to give students and teachers a way to choose Labs by concept, and by search. We expect newcomers to the site to look around, to explore, to try a Lab that looks interesting to them. We later expect students to be more systematic and to choose Labs that fit the topic they are interested in or that they are studying. A website should grab the attention of students and peak their interests to make choice real to them, and I would not be afraid to use the word sell. This visual Explore homepage is a first step in this process by showing the variety of opportunities to enjoy creative problem solving, to see mathematical patterns, to learn important math ideas and skills, to find answers to questions, and to just plain have fun.

Explore lays out Labs in broad categories (numeracy, ratio, functions, finance, statistics, and rate of change) for convenience. As we found earlier the commonly held scope and sequence, we dubbed Leonardo's Math was based on the complexity of the algorithms and not on the difficulty of the concepts. It goes away when we are using digital age tools. We find that like the standard disciplines (mathematics, physics, biology, chemistry, American history, English, art), the strict segregation of the mathematical topics (arithmetic, algebra I, geometry, algebra II, etc.) the silos so long the prominent features of our educational system, are vanishing in the cross-disciplinary digital age world. Likewise, content in our schools will be constantly changing to remain relevant, valuable, and interesting.

Tours



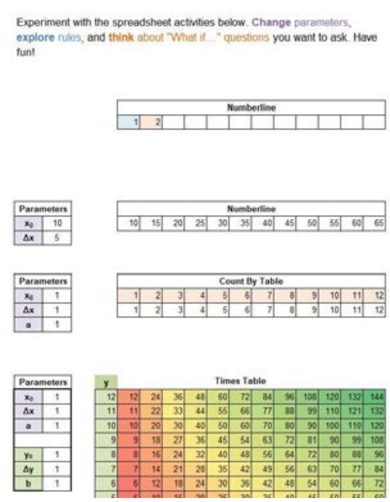
There are essentially two approaches to any learning opportunity. Like a new tourist we can explore a region by picking interesting places to go and see in no ordained order, or we can take a tour, a sequenced and orderly progression. They are different and both are valuable. Often, we take a *tour* on our first visit to a new country and once we have the “lay of the land” we *explore* it ourselves. Tours offer us an overview and a sense of place, history, and general character. Built as a narrative, they tell us a story. Explorations, on the other hand, offer us a chance to learn in more detail and depth something we care about or enjoy. What is true of discovery travel is true of schooling.

Tours of a subject can show it as a whole, to see its unity and progression, to understand it, capture its fundamentals, and navigate it. We were astonished as we began building our first version of what we came to call the Tour. Astonished by the commonality of vision across this expanse of mathematics. For all the major mathematical concepts we want students to learn can be laid out in a common format, differentiated by the parameters we assign and rules we use. Spreadsheets enable an incredible simplification of mathematics even as it wildly broadens its use. Our What if Math Tour is an example of the amazing power of functions[89] and spreadsheet technology to unify our understanding of mathematics and to tell the story of mathematics. Read through these exemplars[90] and then take the real Tour on your live dynamic spreadsheet. I hope you find it as exhilarating as I did and see it as another way digital age technology can replace lectures.

Number lines are tables in columns or rows. We have students construct them functionally with rules to build tables, and in the process learn the fundamental spreadsheet concepts copy/paste and absolute/relative addressing. Then they learn to link numberlines with rules (functions) to build tables with two or more rows or columns.

Parameters are a critical part of any spreadsheet, fundamental to coding and make spreadsheets dynamic and flexible.

Tables can be built in two dimensions like the multiplication table. We build Times Tables from the bottom up, with factors as ordered pairs, so the pattern links to graphing.



[89] It should have come as no surprise for functions rule mathematics today.
[90] These are images from What if Math Tour.

Conditionals – We introduce them early and encourage students to use conditionals (if...then logic) to highlight ideas with color as we have done with the Times Table and the Ratio table.

Parameters	
x_0	1
Δx	1
a	1
y_0	1
Δy	1
b	1

Times Table

y													
12	12	24	36	48	60	72	84	96	108	120	132	144	
11	11	22	33	44	55	66	77	88	99	110	121	132	
10	10	20	30	40	50	60	70	80	90	100	110	120	
9	9	18	27	36	45	54	63	72	81	90	99	108	
8	8	16	24	32	40	48	56	64	72	80	88	96	
7	7	14	21	28	35	42	49	56	63	70	77	84	
6	6	12	18	24	30	36	42	48	54	60	66	72	
5	5	10	15	20	25	30	35	40	45	50	55	60	
4	4	8	12	16	20	24	28	32	36	40	44	48	
3	3	6	9	12	15	18	21	24	27	30	33	36	
2	2	4	6	8	10	12	14	16	18	20	22	24	
1	1	2	3	4	5	6	7	8	9	10	11	12	
*	1	2	3	4	5	6	7	8	9	10	11	12	x

Ratio – Treating multiplication and division in the same table fashion further highlights patterns and mathematics as the "Science of Patterns" opening the door to the wealth of ratio and proportion problems.

Parameters	
x_0	1
Δx	1
p	0.25
y_0	1
Δy	1
b	

Ratio and Proportion Table (with Conditionals)

12												1/1
11											1/1	11/12
10										1/1	10/11	5/6
9									1/1	9/10	9/11	3/4
8								1/1	8/9	4/5	8/11	2/3
7							1/1	7/8	7/9	7/10	7/11	7/12
6						1/1	6/7	3/4	2/3	3/5	6/11	1/2
5					1/1	5/6	5/7	5/8	5/9	1/2	5/11	5/12
4				1/1	4/5	2/3	4/7	1/2	4/9	2/5	4/11	1/3
3			1/1	3/4	3/5	1/2	3/7	3/8	1/3	3/10	3/11	1/4
2		1/1	2/3	1/2	2/5	1/3	2/7	1/4	2/9	1/5	2/11	1/6
1	1/1	1/2	1/3	1/4	1/5	1/6	1/7	1/8	1/9	1/10	1/11	1/12
/	1	2	3	4	5	6	7	8	9	10	11	12

Pascal's Triangle – Tables can be built and explored in a variety of different ways. Make Pascal's Triangle quickly and easily by cutting and pasting a simple addition rule across the cells.

Parameters	
x_0	1
Δx	0

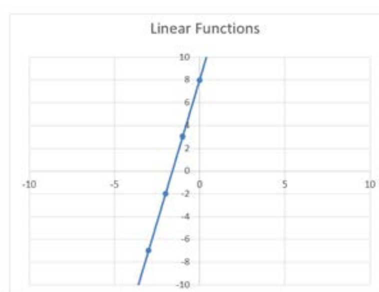
1															
	1														
	1	1													
	1	2	1												
	1	3	3	1											
	1	4	6	4	1										
	1	5	10	10	5	1									
	1	6	15	20	15	6	1								
	1	7	21	35	35	21	7	1							
	1	8	28	56	70	56	28	8	1						
	1	9	36	84	126	126	84	36	9	1					
	1	10	45	120	210	252	210	120	45	10	1				
	1	11	55	165	330	462	462	330	165	55	11	1			
	1	12	66	220	495	792	924	792	495	220	66	12	1		
	1	13	78	286	715	1287	1716	1716	1287	715	286	78	13	1	
	1	14	91	364	1001	2002	3003	3432	3003	2002	1001	364	91	14	1

Linear Functions, not equations, are our prime focus as students learn to picture slope and **y**-intercept by experimenting with parameters.

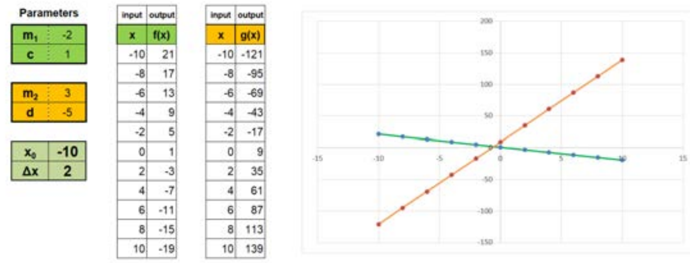
Parameters	
x_0	1
Δx	1
m	5
b	8

f(x)=mx+b

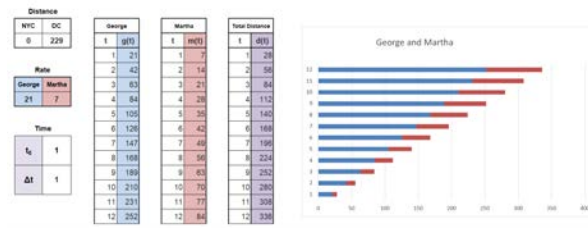
x	f(x)
-5	-17
-4	-12
-3	-7
-2	-2
-1	3
0	8
1	13
2	18
3	23
4	28
5	33
6	38



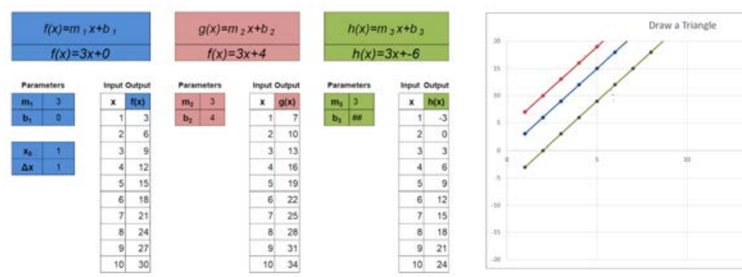
Solving Equations – Equations are the equality of two linear functions.[91] We solve them by finding their point of intersection on the table or graph.



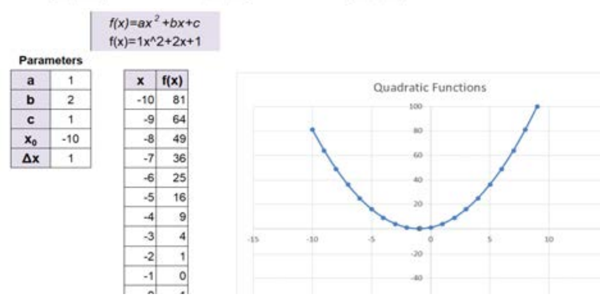
Classic Story Problem – Motion problems like all traditional story problems can follow this same model building 1 table at a time. The first table represents George's motion, the second Martha's motion, and the third table is a rule combining them.



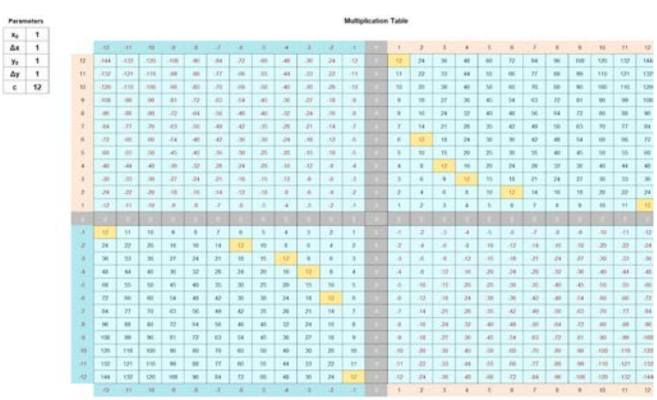
Algebra to build Geometry – Make triangles from these 3 linear functions by changing the parameters of each of the lines to study slope, domain, and range.

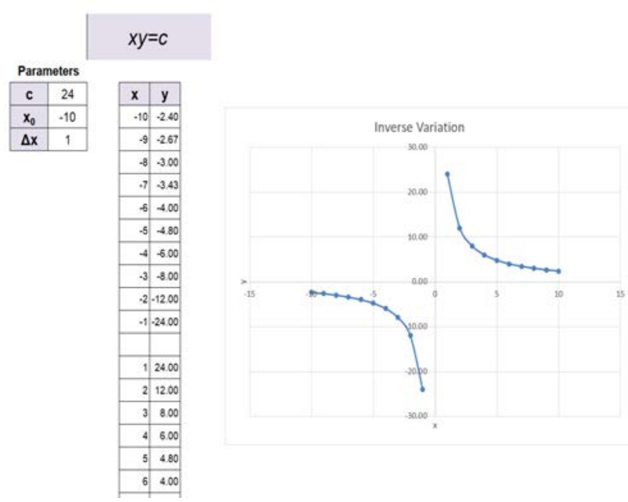


Quadratic Functions – What parameters would you change to flip the parabola, move it up and down, or ask "What does changing '**b**' do?"[92]



Inverse Functions – as viewed graphically and as a multiplication table.





[91] As taught to us all by Judah Schwartz

[92] This question is all too rarely asked in our algebra classes.

Trig functions – In the same way, using different parameters, experiment with other important functions.

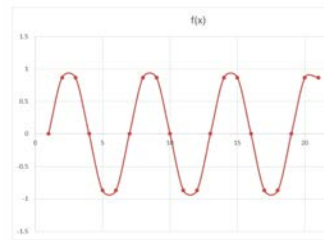
$f(x)=a(\sin(bx+c))$

$f1(x)=1(\sin(2x+0))$

Parameters

Amplitude	a	1
Frequency	b	2
Phase	c	0
Initial input	x_0	0
Increment	Δx	0.52
pi	π	3.14

	x	f(x)
-10	0	0
-9	0.52	0.87
-8	1.05	0.87
-7	1.57	0
-6	2.09	-0.87
-5	2.62	-0.87
-4	3.14	-0
-3	3.67	0.87
-2	4.19	0.87
-1	4.71	0
0	5.24	-0.87
1	5.76	-0.87
2	6.28	0
3	6.81	0.87
4	7.33	0.87
5	7.85	-0
6	8.38	-0.87
7	8.9	-0.87
8	9.42	-0
9	9.95	0.87

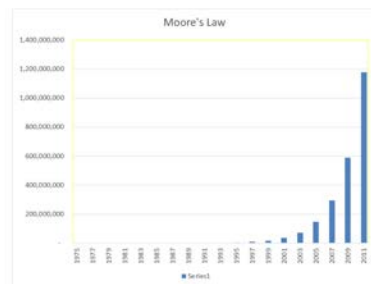


Exponential functions – Experiment with real data and real problems as for example Moore's Law with different rates of "doubling".

Parameters

x_0	1975
Δx	2
Trans	4500

Year	Transistors
1975	4,500
1977	9,000
1979	18,000
1981	36,000
1983	72,000
1985	144,000
1987	288,000
1989	576,000
1991	1,152,000
1993	2,304,000
1995	4,608,000
1997	9,216,000
1999	18,432,000
2001	36,864,000
2003	73,728,000
2005	147,456,000
2007	294,912,000
2009	589,824,000



Compound Interest – Exponential functions applied to simple vs. compound interest where students ask, "Which is fairer?"

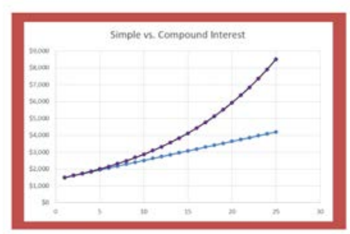
Principal	$1,500.00
Rate	7.5%
t_0	0
Δt	1

Simple Interest

Time	Interest Earned	Balance	Rate
0		$1,500	
1	$113	$1,613	7.0%
2	$113	$1,725	7.0%
3	$113	$1,838	6.5%
4	$113	$1,950	6.1%
5	$113	$2,063	5.8%
6	$113	$2,175	5.5%
7	$113	$2,288	5.2%
8	$113	$2,400	4.9%
9	$113	$2,513	4.7%
10	$113	$2,625	4.5%
11	$113	$2,738	4.3%
12	$113	$2,850	4.1%
13	$113	$2,963	3.9%

Compound Interest

Time	Interest Earned	Balance	Rate
0	$0	$1,500	
1	$113	$1,613	
2	$121	$1,733	7.5%
3	$130	$1,863	7.5%
4	$140	$2,003	7.5%
5	$150	$2,153	7.5%
6	$162	$2,315	7.5%
7	$174	$2,489	7.5%
8	$187	$2,675	7.5%
9	$201	$2,876	7.5%
10	$216	$3,092	7.5%
11	$232	$3,323	7.5%
12	$249	$3,573	7.5%
13	$268	$3,841	7.5%



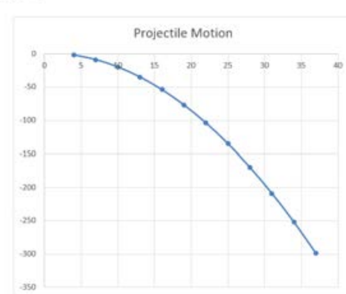
Parametric Equations – Build curves of all kinds by joining functions.

$x = mt + b \quad y = at^2 + bt + c$

Parameters	
t_o	1
Δt	1
Linear	
m	3
b	1
Quadratic	
a	-2
b	-1
c	1

Function Table

t	x	y
1	4	-2
2	7	-9
3	10	-20
4	13	-35
5	16	-54
6	19	-77
7	22	-104
8	25	-135
9	28	-170
10	31	-209
11	34	-252
12	37	-299



Recursion – Use this powerful coding concept to discover new ways of solving quadratic equations.

Parameters	
x_0	3.00
a	1
b	2
c	-1

$x_{n+1}=(-c/(ax_n+b))$

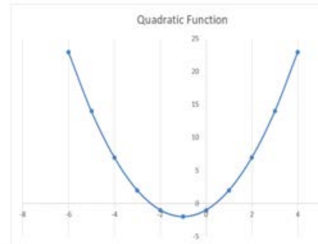
n	x_n
1	3.0000000
2	0.2000000
3	0.4545455
4	0.4074074
5	0.4153846
6	0.4140127
7	0.4142480
8	0.4142077
9	0.4142146
10	0.4142134
11	0.4142136

$x_{n+1}=(-c/(x_n-b))/a$

n	x_n
1	3.00000000
2	-1.6666667
3	-2.6000000
4	-2.3846154
5	-2.4193548
6	-2.4133333
7	-2.4143646
8	-2.4141876
9	-2.4142180
10	-2.4142128
11	-2.4142137

ax^2 +bx +c

x	f(x)
-6	23
-5	14
-4	7
-3	2
-2	-1
-1	-2
0	-1
1	2
2	7
3	14
4	23



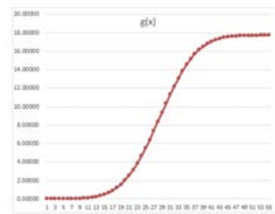
Calculus – Build a table for any function (in this case a Gaussian Distribution). Then sum its values to numerically integrate it and find the difference in its values to differentiate it.

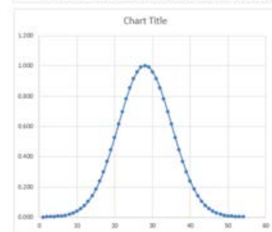
Normal Distribution

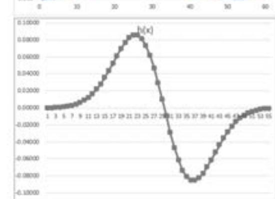
f(x) = e^(-a(x/s)^(2))/2

Parameters	
s	1
x_0	0.5
a	-2.7000
Δx	0.1000

x	f(x)	x	g(x)	x	h(x)
-2.700	0.001	-2.700	0.000	-2.700	0.00000
-2.600	0.001	-2.600	0.001	-2.600	0.00048
-2.500	0.002	-2.500	0.003	-2.500	0.00077
-2.400	0.003	-2.400	0.006	-2.400	0.00122
-2.300	0.005	-2.300	0.011	-2.300	0.00189
-2.200	0.008	-2.200	0.019	-2.200	0.00287
-2.100	0.012	-2.100	0.031	-2.100	0.00425
-2.000	0.018	-2.000	0.050	-2.000	0.00616
-1.900	0.027	-1.900	0.077	-1.900	0.00874
-1.800	0.039	-1.800	0.116	-1.800	0.01211
-1.700	0.056	-1.700	0.171	-1.700	0.01641
-1.600	0.077	-1.600	0.249	-1.600	0.02173
-1.500	0.105	-1.500	0.354	-1.500	0.02809
-1.400	0.141	-1.400	0.495	-1.400	0.03546
-1.300	0.185	-1.300	0.680	-1.300	0.04366
-1.200	0.237	-1.200	0.916	-1.200	0.05241
-1.100	0.298	-1.100	1.215	-1.100	0.06127
-1.000	0.368	-1.000	1.583	-1.000	0.06968
-0.900	0.445	-0.900	2.027	-0.900	0.07698
-0.800	0.527	-0.800	2.555	-0.800	0.08243
-0.700	0.613	-0.700	3.167	-0.700	0.08533
-0.600	0.698	-0.600	3.865	-0.600	0.08505
-0.500	0.779	-0.500	4.644	-0.500	0.08112
-0.400	0.852	-0.400	5.496	-0.400	0.07334
-0.300	0.914	-0.300	6.410	-0.300	0.06179
-0.200	0.961	-0.200	7.371	-0.200	0.04686
-0.100	0.990	-0.100	8.361	-0.100	0.02926
0.000	1.000	0.000	9.361	0.000	0.00995
0.100	0.990	0.100	10.351	0.100	-0.00995
0.200	0.961	0.200	11.312	0.200	-0.02926
0.300	0.914	0.300	12.225	0.300	-0.04686
0.400	0.852	0.400	13.079	0.400	-0.06179
0.500	0.779	0.500	13.856	0.500	-0.07334
0.600	0.698	0.600	14.554	0.600	-0.08112
0.700	0.613	0.700	15.167	0.700	-0.08505
0.800	0.527	0.800	15.694	0.800	-0.08533
0.900	0.445	0.900	16.139	0.900	-0.08243
1.000	0.368	1.000	16.507	1.000	-0.07698
1.100	0.298	1.100	16.805	1.100	-0.06968

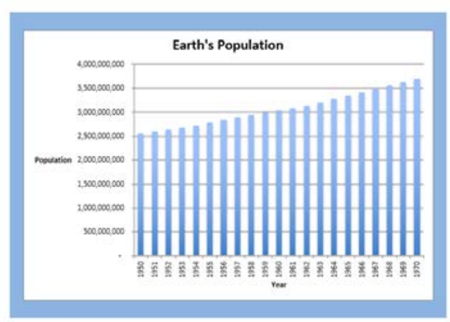






Rate of Change – "Is the earth's population growth speeding up or slowing down?"

Year	Population	Growth
1950	2,556,000,000	
1951	2,594,000,000	
1952	2,636,000,000	
1953	2,681,000,000	
1954	2,729,000,000	
1955	2,780,000,000	
1956	2,833,000,000	
1957	2,889,000,000	
1958	2,945,000,000	
1959	2,997,000,000	
1960	3,039,000,000	
1961	3,080,000,000	
1962	3,136,000,000	
1963	3,205,000,000	
1964	3,276,000,000	
1965	3,345,000,000	
1966	3,416,000,000	
1967	3,485,000,000	



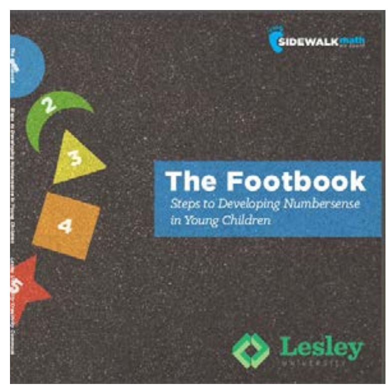
What would a digital age math curriculum look like?

To begin with, it would not be a math curriculum but a STEAM problem-solving curriculum, with the goal of building the creative functional thinking skills every student will need to thrive in this digital age. I imagine these foundational ideas not as a formal sequence but rather a suggested focus that relates grade level to the type of content we traditionally and developmentally have defined. I believe we must focus, always focus, on patterns and patternmaking. I start by encouraging kids to build counting patterns, the foundation of numbersense.

PK-2 Numbersense by Counting (Sidewalk Math[93])

Sidewalk Math in Haiti

We generally believe that numbersense, in its biggest sense, is the mathematics mission of education.[94] It defines whether we get and can use math, and whether we understand the patterns of math; for the rest is really a matter of following the rules. Since mathematics is the science of patterns, in a fundamental way it is the science of numbersense. So, any digital age math curriculum must begin with a focus on numbersense, on the patterns of numbers and their operations to build headmath and handmath on digital age devices.



A few years ago, while enjoying another Thai lunch and staring out the window of my favorite restaurant, I began to wonder whether we could get young children to build numbersense as they walked down a busy street with their caregivers. Kids love hopscotch which has unfortunately no interesting mathematical pattern. What if we painted, I wondered, useful numbersense oriented math patterns on the sidewalks for kids to play and dance on? We named it Sidewalk Math, brought it to Lesley University's Creativity Commons[95] and found a brilliant design student to make the patterns beautiful and usable to help children everywhere learn numbersense through active play to prepare them for learning math and patternmaking when they get to school.

I favor thinking about numbersense as all about counting. All our Sidewalk Math patterns are designed to encourage kids to move and count, and to encourage their parents and caregivers

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93 Sidewalk Math (http://www.sidewalkmath.org) Lesley University Creativity Commons
94 The data is unfortunately clear. Enter school with numbersense and you succeed in math and in reading as well, without it you likely will struggle. Numbersense is a "you know it when you see it" quality.
95 Creativity Commons was founded and run by Professor Martha McKenna.

Sidewalk Math -- Count the Dragon

to count with them all the time and with as many different things and in as many different ways as possible. Counting corners as in the Dragon builds both counting objects to 12, counting corners in common shapes, and adding numbers. Counting By is designed not only to count by 1's, but also by 2's, 3's, 5's, and 10's to build their multiplication facts. In each of these designs, the emphasis for PK - 3 should be on making counting both a mental (headmath) activity and a physical activity.

Whether you paint sidewalks, make patterns using chalk, or purchase carpets[96] with these patterns for your school classrooms, make building numbersense a daily core physical activity. I encourage kids, parents, and teachers to develop their own activities, both these large-scale ones and small-scale manipulatives ones. *We Count* should be the focus of early schooling and the theme of all communities building numbersense and ensuring all kids are ready to use math and quantitative reasoning to thrive in the digital age.

Sidewalk Math -- Count By (building multiplication facts)

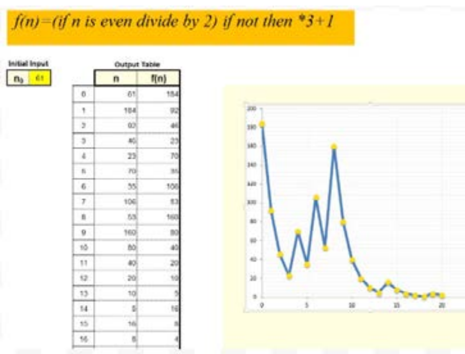
1-3 Numbersense on Spreadsheets

Multiplication Table

12	12	24	36	48	60	72	84	96	108	120	132	144
11	11	22	33	44	55	66	77	88	99	110	121	132
10	10	20	30	40	50	60	70	80	90	100	110	120
9	9	18	27	36	45	54	63	72	81	90	99	108
8	8	16	24	32	40	48	56	64	72	80	88	96
7	7	14	21	28	35	42	49	56	63	70	77	84
6	6	12	18	24	30	36	42	48	54	60	66	72
5	5	10	15	20	25	30	35	40	45	50	55	60
4	4	8	12	16	20	24	28	32	36	40	44	48
3	3	6	9	12	15	18	21	24	27	30	33	36
2	2	4	6	8	10	12	14	16	18	20	22	24
1	1	2	3	4	5	6	7	8	9	10	11	12
*	1	2	3	4	5	6	7	8	9	10	11	12

When we build numbersense on spreadsheets with numberlines and tables using rules, we can ask students to do interesting things. For example: "What rule could you use to build a numberline of just the odd numbers?" "What rule would you use to make a table of the products from 1 to 12?" We can ask questions we have not before asked of our students like, "How many of the products in a 12 by 12 times table are odd numbers, and why is that important to know?" "Find a rule that will build 10's tables or 100's tables, or subtraction tables." "Make a place value machine or run a store."

[96] Sidewalk Math carpets are manufactured by Flagship Carpets available through school suppliers.



The Syracuse Problem

Spreadsheets enable students to explore sequences of numbers, do they converge or diverge? For example, the Syracuse problem asks, what is the result of picking a number, any number: if it is even, divide it by 2; and if it is odd, multiply it by 3 and add 1? Do the same to the result and so on… "What will you end up with?" "Is that true for all whole numbers?" Spreadsheets, by taking the pain out of repeated calculation and by enabling students to use rules to repeat processes, make repeating this problem for many different numbers, or playing with a wide variety of similar ones, makes numbersense physical, fun, and creative for kids.

4-6 Ratio

Ratio and Proportion Table

12	12/1	12/2	12/3	12/4	12/5	12/6	12/7	12/8	12/9	12/10	12/11	12/12
11	11/1	11/2	11/3	11/4	11/5	11/6	11/7	11/8	11/9	11/10	11/11	11/12
10	10/1	10/2	10/3	10/4	10/5	10/6	10/7	10/8	10/9	10/10	10/11	10/12
9	9/1	9/2	9/3	9/4	9/5	9/6	9/7	9/8	9/9	9/10	9/11	9/12
8	8/1	8/2	8/3	8/4	8/5	8/6	8/7	8/8	8/9	8/10	8/11	8/12
7	7/1	7/2	7/3	7/4	7/5	7/6	7/7	7/8	7/9	7/10	7/11	7/12
6	6/1	6/2	6/3	6/4	6/5	6/6	6/7	6/8	6/9	6/10	6/11	6/12
5	5/1	5/2	5/3	5/4	5/5	5/6	5/7	5/8	5/9	5/10	5/11	5/12
4	4/1	4/2	4/3	4/4	4/5	4/6	4/7	4/8	4/9	4/10	4/11	4/12
3	3/1	3/2	3/3	3/4	3/5	3/6	3/7	3/8	3/9	3/10	3/11	3/12
2	2/1	2/2	2/3	2/4	2/5	2/6	2/7	2/8	2/9	2/10	2/11	2/12
1	1/1	1/2	1/3	1/4	1/5	1/6	1/7	1/8	1/9	1/10	1/11	1/12
/	1	2	3	4	5	6	7	8	9	10	11	12

Most of the quantitative problem solving we do in our work and daily lives involves ratios. Ratios are the patterns produced by the division of two quantities. A rate like kilometers/hour is a ratio, as is percentage, interest, proportions of all kinds, conversions, as well as fractions, decimals, and most sports statistics like batting average. The traditional math curriculum treats each of these varieties of ratios as a separate concept, siloed and focused on a particular type of problem. Since students rarely encounter ratio as a general concept, they learn to solve problems with ratios separately and mechanically, memorizing each as its own process, and practice applying each of these algorithms to its own niche.

Imagine instead, using spreadsheets to solve ratio problems. Change the form of the ratio by simply changing the format of the cell.

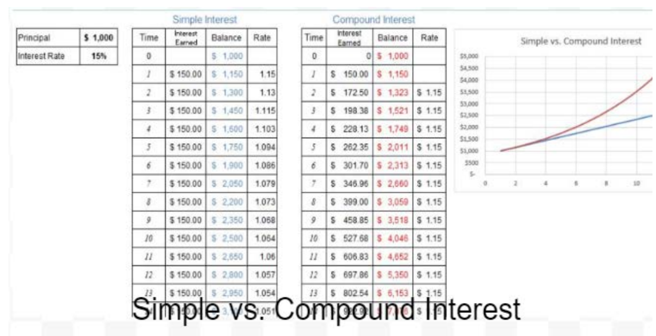
Parameter Table	
Numerator-i	1
Denominator-i	7
Δx	1
x_o	1

x	Numerator	Denominator	Ratio	Fraction	Decimal	Percents	Money
1	1	7	1/7	1/7	0.143	14%	$0.14
2	2	7	2/7	2/7	0.286	29%	$0.29
3	3	7	3/7	3/7	0.429	43%	$0.43
4	4	7	4/7	4/7	0.571	57%	$0.57
5	5	7	5/7	5/7	0.714	71%	$0.71
6	6	7	6/7	6/7	0.857	86%	$0.86
7	7	7	7/7	1	1.000	100%	$1.00
8	8	7	8/7	1 1/7	1.143	114%	$1.14
9	9	7	9/7	1 2/7	1.286	129%	$1.29
10	10	7	10/7	1 3/7	1.429	143%	$1.43
11	11	7	11/7	1 4/7	1.571	157%	$1.57
12	12	7	12/7	1 5/7	1.714	171%	$1.71
13	13	7	13/7	1 6/7	1.857	186%	$1.86
14	14	7	14/7	2	2.000	200%	$2.00
15	15	7	15/7	2 1/7	2.143	214%	$2.14

Change a decimal to a percent by clicking on the % button. Change it to a fraction by formatting it as a fraction. Convert centigrade to Fahrenheit by choosing a formula, or plot proportions as a linear function with a straight-line graph. Indeed, linear functions are ratios too. Ratios are quantities that generally have wide, often infinite, variety (proportions). They are usually functions, with a rule connecting their numerator as an input to a unique denominator as the output or vice versa. By changing the rule, the inputs, or the outputs, or by changing the formatting, students can play with ratios and use them to solve the wide variety of problems they meet.

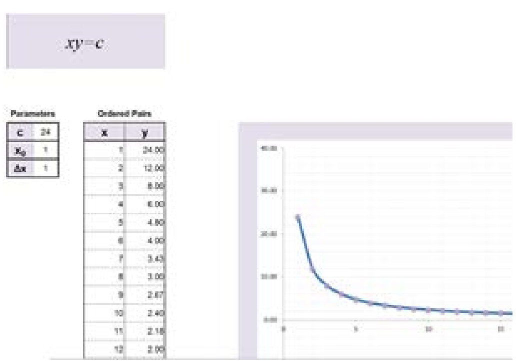
Visualize ratios as tables with two columns or rows of values, as graphs with a straight sequence of points or a line, or even as lines of cells in a table. Spreadsheets, by automatically linking tables and graphs, enable students to see how the slope changes as the ratio changes. This all-important connection, central to linear functions and fundamental to calculus as rate of change is another example of how this technology enables students to turn concrete instances into abstract concepts and to build their powers of visualization.

Ratio even includes common statistics; probability and mean are both ratios. As central to the 4th through 7th grade curriculum, ratio would enable us to build financial reasoning with topics like: compound interest, investment, loans and credit cards, auto finance, home finance, small business finance, personal finance, contractor finance, government finance, lottery, cost of living… which should be critical topics to develop financial problem-solving ability and managerial expertise.



Simple vs. Compound Interest

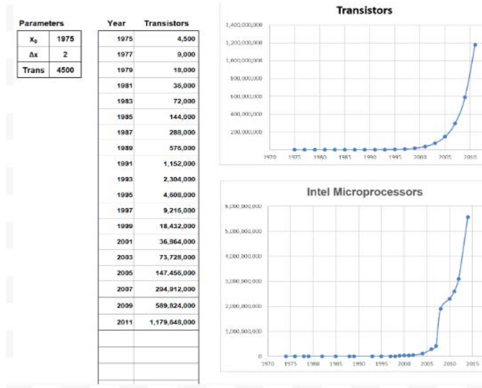
6-12 Functions and Model-Building



Functions, generally considered the most important idea in mathematics, should be , central as expected to any math program. As powerful function machines, spreadsheets enable students to create and explore functions: linear, systems, families, operations, rate of change, and more. Spreadsheets make it easy to visualize functions to link and explore them as symbols, tables, graphs, and even animations, to picture them, change them, explore their nature, and predict their consequences. Is climate change linear, exponential, or some other function? Is population growth slowing down or speeding up? Is GDP a good measure of economic health?

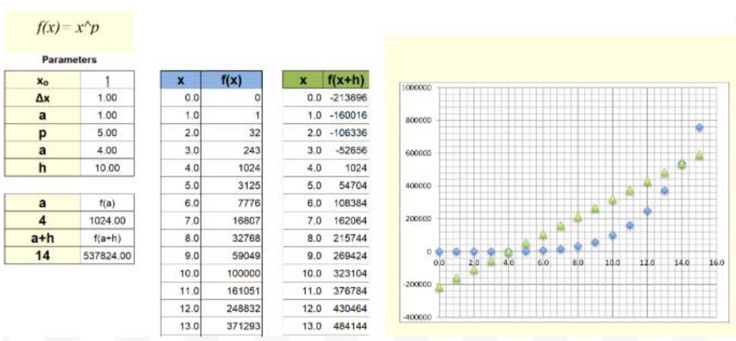
Functions are not only important as fundamental ideas in mathematics, they are essential to understanding scientific reasoning, experimental data, and asking the creative “What if…” They are the key concept in coding, transforming inputs into outputs, if…then conditional statements, iteration, and even recursion in which a function’s output becomes its input. This is why spreadsheets are superb platforms for learning, practicing, and understanding coding. They enable students to develop functional thinking as a way of problem solving, for functions involve visualization, organizing data into tables, building models by combining functions, iterating those models, and then asking the exploratory question, “What if…”

Functions are central to science as well as to technology. They are the way we model cause and effect. Their basic form matches the process of experimental science. An experiment takes some input, applies a well-defined process to that input creating a unique output. And functional thinking in terms of inputs, outputs, and rules are central to engineering processes as well. We expect to build machines that take inputs and transform them into outputs, whether we are building, manufacturing, or testing a new product or process. Functions are the very foundation of the STEM disciplines. And the digital age is the age of functions, especially discrete functions. They have become our primary tool for solving problems, visualizing and modeling our world. They are critical for every one of our students to understand and comfortably work with.



What if Math -- Moore’s Law

8-12 Rate of Change



Newton’s Quotient

Why isn’t calculus for everyone? Though calculus has broadly penetrated high schools because of Advanced Placement,[97] it is still considered abstract, difficult, and only for the academically talented. But while the curriculum sequence has left **∆x** outside the mainstream of math content, I don’t think that is the primary reason we do not include it in every student’s education from an early age. We do not find ∆**x** in the standard math curriculum because the algebra we teach is analog, the algebra of continuous variables. We think of **x** as continuous and our students are supposed to learn to solve for it or manipulate expressions involving it. We skip over the discrete ∆**x** and jump right into the continuous, more abstract, and much harder than the infinitesimal **dx** of differential calculus. We give students no foundation for thinking about and expressing rate of change, so

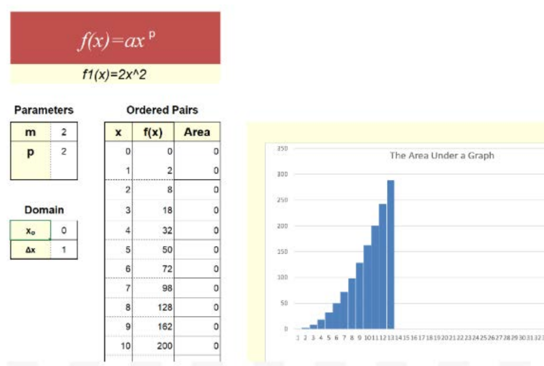
97 Approximately 10% of the students in each high school year take an AP Calculus course. www.ericdigests.org/pre-9217/calculus.htm

fundamental in our STEM world, forcing them to learn calculus mechanically, as a collection of algorithms for working with abstract infinitesimals.

With the advent of computers and spreadsheets, we now treat most real-world math digitally, focusing on discrete variables, finding key values in tables and displaying their forms in graphs. Traditional school algebra clings to those functions that are solvable and uses tables of values generated by those functions to graph them. It thus goes from the abstract to the concrete, just in the opposite direction we normally build and understand concepts. If **x** represents a filled in number line, then ∆**x** represents a section of that line. We have been forced to focus on continuous variables because math education primarily relies on analog (paper algorithm-based) calculation and the solving of equations. We have no need for ∆**x** in most of our existing curriculum. And shrinking ∆**x** by using limits is very abstract. It baffled me when I had to learn it and continues to confuse students. Our digital age works with discrete variables to solve problems involving discrete data.

Discrete variables and functions make ∆**x** an inherent and even critical part of their definition. It is the parameter we use from the get-go to increment the independent variable. It enables students to control the accuracy of the solution and the scale of a table or a graph.[98] It focuses on using rules to generate and control domain and range elevating them to the status they deserve as inherent in the actual definition of function. And for the first time it brings the critical ideas of the calculus, rate of change and total change[99] to all students. In the digital we all have to understand and deal with rate of change with questions like:

- "Is global warming speeding up or slowing down?"
- "At the rate we are spending, will our savings last our lifetime?"
- "What is the point of inflection, when will we start making money on this new product?"
- "When will we reach maximum size?"
- "What is the total change we can expect?"[100]

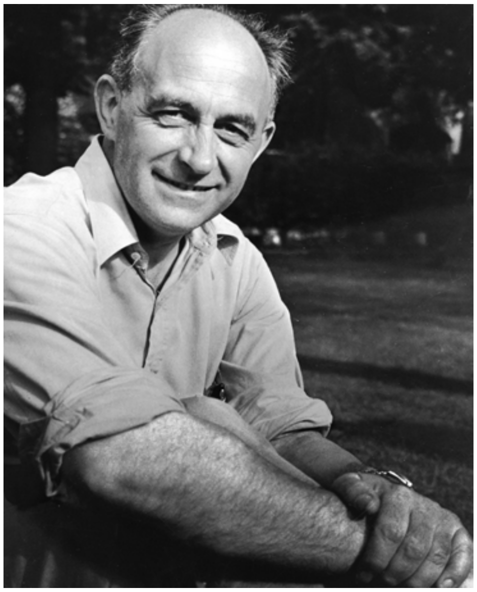


Area under a curve

If you want to help students become better problem solvers in this digital age, introduce **x** with discrete variables and **∆x** as the discrete steps between the values of **x**. Enable them to become familiar with **∆x** changing its value, shrinking it like zooming in, us it to visualize slope on a curve as a ratio, and work conceptually and digitally with derivatives and integrals.

[98] I am grateful to Ryan McQuade for this insight
[99] Calculus is about a pair of functions. (Gilbert Strang video lectures on calculus)
[100] The image illustrates the area under a curve as a Riemann sum of thinner and thinner rectangles produced by shrinking **∆x.**

Fermi Problems–Headmath Throughout



Of the 35 or so Nobel Laureates in physics affiliated with the University of Chicago, likely more than any other university, one stands out, one is considered the most brilliant, one has stories told about him, Enrico Fermi. As a student in physics we were told Fermi stories. He would go into the physics library, pick up the latest journal, turn to its back-cover table of contents. If he found something of interest, he would solve the problem himself and then turn to the article to check the author's work. Fermi was revered and considered by other great physicists the very brightest of them all.

Today, most people who have heard of Enrico Fermi think of Fermi problems, estimation problems which can be solved mentally like the classic, "How many piano tuners are there in the City of Chicago?" Philip Morrison, a giant physicist in his own right, described them as:

> *... the estimation of rough but quantitative answers to unexpected questions about many aspects of the natural world. The method was the common and frequently amusing practice of Enrico Fermi, perhaps the most widely creative physicist of our times. Fermi delighted to think up and at once to discuss and to answer questions which drew upon deep understanding of the world, upon everyday experience, and upon the ability to make rough approximations, inspired guesses, and statistical estimates from very little data.*[101]

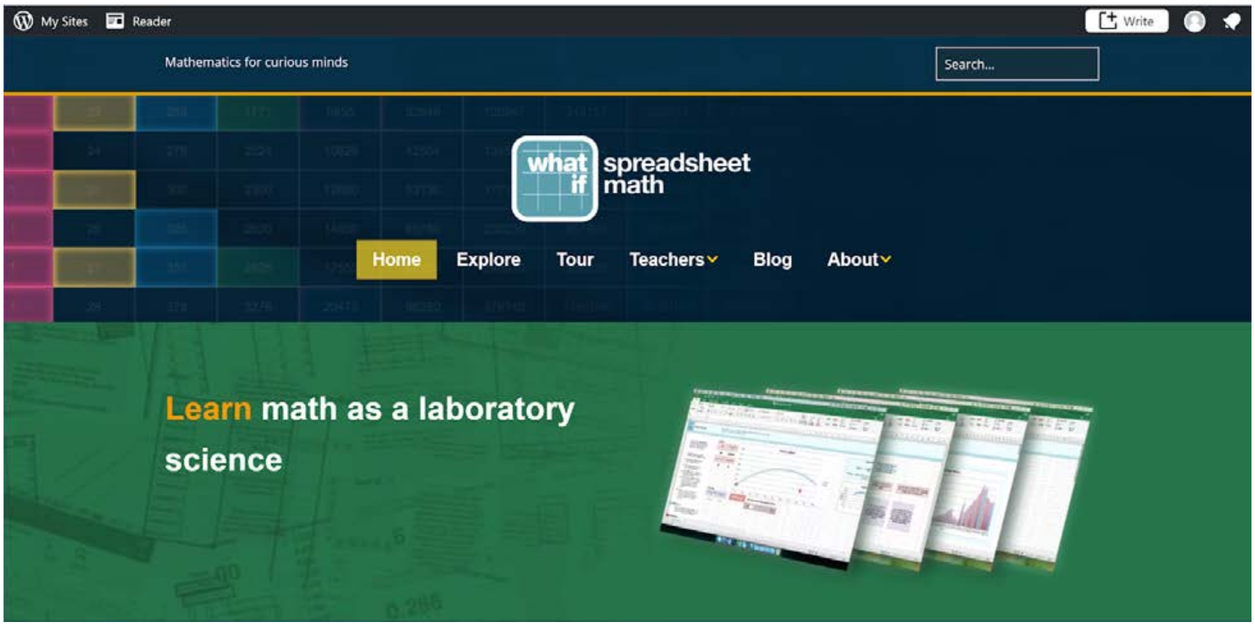
Fermi problems are *headmath* problems, the calculations we do and can do in our minds. Headmath is not algorithmic, there is no standard process to be slavishly followed. It is a creative process. We simplify the problem and seek, as Morrison says, approximations and estimations. While most of the real-world problems we solve daily do not rise to the level we would call Fermi problems, they are solved in the same way. For example, to calculate a 15% tip on a $45 dinner, I think 1/10th and then half of that $4.50 + $2.25 which I round up to $7.00 to be a little more generous. I don't need a calculator or a pencil and paper, all I need is a little headmath time and practice.

Whether we need to calculate tips, know if a business deal we just heard about is worth pursuing, want to argue with some crazy number we heard on TV, or solve a Fermi problem in a job interview at Google; we are practicing headmath and pursuing mathematics estimation skills

[101] Philip Morrison, Letters to the Editor, Am. J. Phys., August 1963, v31n8 p626-627.

critical to problem solving in the digital age even though we are carrying around computers in our pockets. **Headmath should be a significant part of our STEAM curriculum, and it should be practiced every single day in our math and STEM classrooms.** It is a natural part of the dimensional analysis every physics student learns thanks to Fermi, and it should be a natural part of our 21st century schools. Just because we are in the digital age does not mean our students are staring at screens all day. It does mean that we must focus on those essential skills our kids will need for their age. And among them are the headmath skills Enrico Fermi taught us to use.

6. What If…



"What if..." the question that experiments enabled science to ask and spreadsheets enabled business to ask, can now become the question every student can ask. Learning can now be joined to creativity. Our kids can view learning as a creative experience in the fullest sense of those words, to enable them to thrive in this new digital age.

Core Values


In CONGRESS. July 4, 1776.

The unanimous Declaration of the thirteen united States of America.

Successful enterprises focus on values, for values are the secret sauce that creates community, commitment, and customers. Our core values as a nation, particularly those defined by Thomas Jefferson in our founding document, shape our country and our society.

We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable rights, that among these are Life, Liberty, and the pursuit of Happiness.[102]

Whether a nation or a high-tech company, the values we espouse and communicate are central to success. Here is how one business magazine article puts it.

Core values are what support the vision, shape the culture and reflect what the company values. They are the essence of the company's identity the principles, beliefs or philosophy of values.[103]

What is true for the nation and for business is true for schools. The core values we define and support with our practice are our identity. If our core value is to achieve higher scores on standardized test then that is the lesson we are teaching, the lesson our kids will learn. If our core value is to prepare our kids to learn how to learn, then their lessons will be quite different. Schools always must ask themselves:

- **What do you think the core values of education in the digital age should be?**
- **What do we really want our kids to learn in school?**

I ask you to think about these questions. We do that far too infrequently. All too often accepting what we are given with little thought to its impact on our students' futures. We accept what we grew up with, instead of what they must grow to. We accept what exists, for fear that what does not, will be worse. Today, it seems that everything new which enters the education space must have "scientific" validity, be tested in a truly objective scientific experiment, and by experiment prove itself better than what now exists. Why?

What now exists in most of our classrooms, the textbooks, the paper worksheets, the teacher presentations, the curriculum, have not been proven to work! The Department of Education in

[102] Declaration of Independence, July 2, 1776

[103] https://thinkmarketingmagazine.com/apple-core-values/

Massachusetts, considered generally as the top education state, just revised its Mathematics Common Core Standards as part of a revision of its MCAS tests. Did they use the wealth of data on student success and failure on the tests given over the past 20 years? Did they ask whether the test results verified the scope and sequence of the curriculum? Did they ask whether the math students are required to learn is appropriate for their age level or developmental stage? They did not. They asked a small group of mathematics educators, principally from the college ranks, to go over each of the standards to make sure they were clearly stated and sensibly positioned. They fundamentally accepted without question almost all the traditional mathematics our kids will have to master during their school years. They took no significant notice of technology or of usefulness. They took no notice of the mathematics our kids will need for their work and life future.[104]

They, one more time, reworked in minor ways a curriculum that was created well over a century ago, for an entirely different purpose. And we know, we have in fact existential proof, that this curriculum, this scope and sequence, fails 60% and more of our kids. We are now running and have been running for more than a century a grand experiment. We have the results. We don't need another so called "scientific" study or any study. When our core values are to conserve and preserve the system that worked to produce skilled and unskilled workers for repetitive jobs, now cranked up to the max to ready all for college and careers, we can fully understand its failure.

Without a conversation about our core values, without a vision of where we want to go and what we want our kids to become, we will continue being lost in ever turning epicycles of blame, disappointment, and failure. It seems to me that the core values Jefferson defined for our nation ought to be the core values for our education: that all our kids are created equal and have the unalienable right to a college education and college degree so they may thrive in the digital age. We must ask ourselves and our educators, "What if our schools practiced our core values and did not just preach them or try to teach them?"

[104] In the Mathematics Common Core document that has been widely distributed, the word ratio(s) appears 12 times while the word fraction(s) appears 200+ times and the word function appear only a dozen times. Now, I ask you, "Which of these math ideas will be more important to our kids?"

Technology Created it, Technology Will Solve it

Most of the problems we face in education today have been caused by technology. Technology has made the world “flat” as Tom Friedman describes its effect on transportation and manufacturing, it has changed healthcare with amazing new pharmaceuticals and treatments that have made medicine and its costs central to the public debate, and it profoundly reshaped business with digital machines taking over repetitive jobs. The last in particular, the digital age, has changed our jobs and is reshaping our economy. It is forcing us to reinvent education if we are to survive as a nation.

These technologies are impacting our schools. We are once again, “a nation of immigrants” with the need to prepare children born and not born here with a higher level of learning, foreign to most of us. Technology has changed the requirements for the jobs of their future, the STEM world of business, have impacted our schools by making new demands on both teachers, students, and administrators. Technology has already been adopted in most of our schools by most of our teachers in some major ways.

Teachers, students and schools are using email to communicate with students, PowerPoint to present ideas, Excel to manage grades, and student information systems like Google Docs and Blackboard to organize and manage student work. They are on Facebook and Twitter, add photos to Snapchat and Instagram, and their cell phones are ubiquitous. Perhaps most important for both teachers and students, the Web has become their library, their encyclopedia, and their primary information resource. All this technology has not produced the promised benefits. Instead of helping teachers and school administrators become more efficient, instead of helping most students learn more and learn better, it has increased their workload coping with 24/7 email communication, overwhelming quantities of data, and disinterested students who no longer find school relevant because they carry a pocket communicator/computer in their pocket.

George Blakeslee has been dealing with these issues for nearly 30 years. He taught in the Technology and Education program at Lesley University, mainly online or in non-traditional classrooms. He recognized that technology would either overwhelm him or he would have to figure out ways to make it support him. To deal with the student emails flooding his inbox he developed the Assignment Receipt Checklist, an online spreadsheet that would tell students that he had received the assigned work and whether it was adequate. He created templates for his support emails on student writing, work that he could then customize to give students valuable but appropriate feedback. And he would capture a collection of student exemplars to give other students working on an assignment a model by which they could create and check their own work, reducing his burden.

Fall Term 1 2012	ECOMP 6016.81	Online	Oct 28 R - received; Cr - complete pending DB replies; C - complete; W - work in progress; Wc - pending comments; e - extension; Op -										
Work Period	Due Date	task	JA	SC	AF	KH	TI	Ska	Ski	JN	CR	SS	GT
Cluster A	**Cluster A - 100 pts**	**Illustrated Posters participation 20**	C	C	C	C	C	C	C	C	C	C	C
Sep 3 - 9	9-Sep	Task 011 Biographical Sketch 5 pts	C	C	C	C	C	C	C	C	C	C	C
		Task 012 CONFU Guidelines & Worksheet 5 pts	C	C	C	C	C	C	C	C	C	C	C
		Task 013 Layout Scenic 5 pts	C	C	C	C	C	C	C	C	C	C	C
		Task 014 Layout Illustration 5 pts	C	C	C	C	C	C	C	C	C	C	C
		Task 015 Layout Golden Spiral 5 pts	C	C	C	C	C	C	C	C	C	C	C
		Task 016 Poster Planning & Discussion (c) 15 pts	C	C	C	C	C	C	C	C	C	C	C
		Task 017 convert to JPG 20 pts	C	C	C	C	C	C	C	C	C	C	C
		Task 018 Create Blog posting Task 016 20 pts	C	C	C	C	C	C	C	C	C	C	C
Cluster B	**Cluster B - 100 pts**	**Editorial Images participation 20 pts**	C	C	C	C	C	C	C	C	C	C	C
Sep 10 - 16	16-Sep	Task 021 Screen capture 10 pts	C	C	C	C	C	C	C	C	C	C	C
		Task 022 (op) Scanning Images (5 pts) =	C	C	C	C	C	C	C	C	=	=	C
		Task 023 (op) Digital Cameras (5 pts) =	C	C	C	C	C	C	C	C	C	=	C
		Task 024 Image Editing Resize 10 pts	C	C	C	C	C	C	C	C	C	C	C
		Task 025 Image Editing Crop 10 pts	C	C	C	C	C	C	C	C	C	C	C
		Task 026 Image Editing Rotate 10 pts	C	C	C	C	C	C	C	C	C	C	C
		Task 027 Image Editing Replace 10 pts	C	C	C	C	C	C	C	C	C	C	C
		Task 028 Edited Image Analysis 10 pts	C	C	C	C	C	C	C	C	C	C	C
		Task 029 Editorial Image Production 10 pts	C	C	C	C	C	C	C	C	C	C	C
		Task 0210 Create Blog posting Task 029 10 pts	C	C	C	C	C	C	C	C	C	C	C
Cluster C	**Cluster C - 50 pts**	**Animation participation 20**	C	C	C	C	C	C	C	C	C	C	C
Cluster G	**Cluster G - 35 pts**	**Read/Discuss 10 pts**	C	C	C	C	C	C	C	C	C	C	C
Sep 17 - 23	23-Sep	Task 041 Animated GIF Crazy Clock 15 pts	C	C	C	C	C	C	C	C	C	C	C
		Task 042 Animate GIF Logo 15 pts	C	C	C	C	C	C	C	C	C	C	C
		Task 071 Read/Discuss Textbook (G) 25 pts	C	C	C	C	C	C	C	C	C	C	C
Cluster D	**Cluster D - 50 pts**	**JING participation 20**	C		C	C	C	C	C	C	C	C	C
Cluster G	**Cluster G - 25 pts**	**Read/Discuss 10 pts**	C		C	C	C	C	C	C	C	C	C
Sep 24 - 30	30-Sep	Task 051 JING tutorial & Script 20pts	C	We	C	C	C	C	C	C	C	C	C
		Task 052 Create Blog posting Task 051 10 pts	C	e	C	C	C	C	C	C	C	C	C
		Task 072 Read/Discuss Multimedia research (G) 15 pts	C	C	C	C	C	C	C	C	C	C	C
Cluster E	**Cluster E - 100 pts**	**Sound participation 20 pts**			C			C	C		C		C
Oct 1 - 7	7-Oct	Task 031 Sound: Audio CDs 10 pts	C	C	C	C	C	C	C	C	C	C	C
		Task 032 Sound: Microphone 10 pts	C	C	C	C	C	C	C	C	C	C	C
		Task 033 Sound: Editing 10 pts	C	C	C	C	C	C	C	C	C	C	C
		Task 034 Sound: Mixing 10 pts	C	C	C	C	C	C	C	C	C	C	C
Oct 8 - 14	14-Oct	Task 035 Sound: Audio Program & script 20 pts	C	C	C	C	C	C	C	C	C	C	C
		Task 037 Create Podcast posting Task 038 MP3 20 pts	C	C	C	C	C	C	C		C	C	C
Cluster F	**Cluster F - 100 pts**	**Video participation 20**			C			C	C		C		C
Cluster G	**Cluster G - 40 pts**	**Reflection 10 pts**			C			C	C		C		C
Oct 15 - 21	21-Oct	Task 061 Analysis: Thematic Sequence compare 10 pts	C	C	C	C	C	C	C	C	C	C	C
		Task 062 Video Planing (Rationale) 10 pts	C	C	C	C	C	C	C	C	C	C	C
		Task 063 Video Planing (Thematic Sequence) 10 pts	C	C	C	C	C	C	C	C	C	C	C
Oct 22 - 28	28-Oct	Task 064 Video Production (Authoring) 25 pts	W	C	C	W	C	C	C	C	C	C	C
		Task 065 Create Blog posting Task 064 25 pts	W	C	C	W	W	C	C		C	W	C
		Task 073 Final Paper (G) 30 pts	R	R	C	R	R	C	C	R	C	R	C

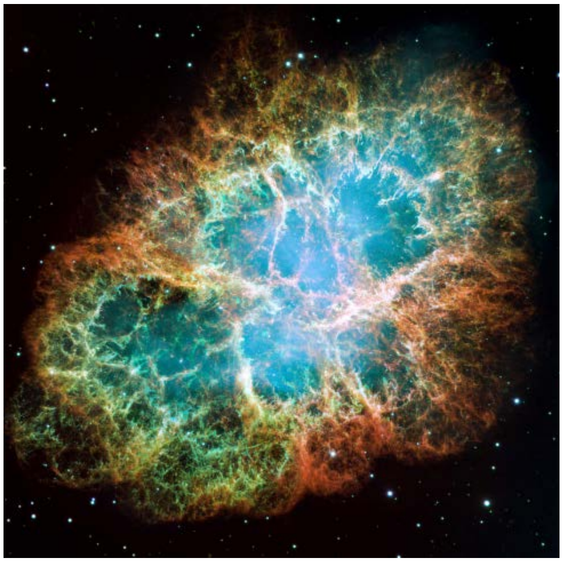
George’s student assignment management spreadsheet

When students asked him for permission to use the tools of their choice, at first, he rebelled because he did not know or know well those tools, but gradually he learned to trust students and let them support themselves on the tools of their choice. He thus lifted the new and constant burden of keeping up with new technology and 24/7 communication and grading of student work and began to focus on the end product giving students more and more responsibility for the learning process. He defined the problem or product students were to solve or make, the “destination”, and put them in charge of getting there. From his point of view, he was starting to think about his courses from the end and working his way back to the beginning.

He rethought the teacher’s and student’s roles. No longer was he responsible for building the learning scaffold, he gave the job to the student. No longer did he present and then grade students on what they passively acquired, he set the target and mentored them as they found their own way to it. No longer did he have to be the know-all of the technology or of all the knowledge, he questioned, encouraged, and coached students to keep at it until they were proud of their solution. He let technology in the hands of students do the work so that he could be the human support and reward they find so valuable.

He did more than use technology to be mechanically more efficient. He recognized that he had to change both what and how he was teaching. He used technology to define a new model. We have seen the outline of this model in *What if Math*. We can apply it to the rest of schooling and think about it not just as a new interactive format for curriculum, a new hybrid classroom, or a new way to package instruction but as the integration of curriculum and instruction.

The Curriculum of Tomorrow



If you are interested in astronomy and it is hard not to find this subject fascinating today, then I suggest you go to https://openstax.org/details/astronomy. This astronomy book by Andrew Fraknoi is as beautiful as any introductory textbook on this subject and it is free, yes **free**! It is just one example of the "textbooks" available on openstax.org. You can get it in most any form, though I imagine most would print it out as a pdf. It is just one example of the way the curriculum of the future will be distributed.

Now, I believe that textbooks, as their name implies, are representatives of the past technology, but we will likely use its format for years to come. Though these online versions take advantage of technology for distribution and often use links to connect what was before found in footnotes or suggested activities at the end of chapters, they remain in form and function inert curriculum defined by a paper permanence past. They take little advantage of digital media or digital tools. They are not rich in links, in multimedia, or in interaction. They are still mainly about imparting knowledge and not about developing skills, about teaching and not about learning. Grounded in text, they are still barriers for many students who would love to learn a subject but cannot read or write sufficiently well to conquer the content at a suitable pace. And most important they are static not dynamic.

As you have seen throughout this work, I look to the What if Math, project as one model for this future. I believe the curriculum of the future will resemble web screens where students build 21st century skills interactively using 21st century tools. They will learn in the world they live, where data is rich and problems varied, where support is just a Google away, where feedback comes from the tools they use or the virtual or real colleagues they work with, and where there is a constant stream of new "cases" to choose. Some will be in the format of problem-based-learning and some of it bigger and more open as project-based-learning. The essential case study format remains.

Whatever form content takes, I believe a significant portion of it will be offered for free or at low costs. The Internet has made knowledge and the acquisition of many skills free and varied. Unlike the very limited range of content and contexts we have today, digital age curricula will look more like an app store, a giant library of cases and problems from which students can find ones that turn them on, fulfill their interests, and tickle their dreams. I expect some content, likely lessons requiring complex development like business school simulations, may well be charged for. But even those will likely be more commodity or hobby creation then planned projects.

The curriculum materials of the future will be profoundly different from the curriculum materials we think of as normal today, the stuff of our paper past. I look forward to the varied forms and wealth of ideas that our kids will be able to learn from in the schools of the digital age. I look forward to content that is much sparser in words and facts than our obese textbooks suggest today. And I look forward to content that combines curriculum and instruction into a single unified tool that all students can use to learn, what they want to learn, when they want to learn it.

The Teacher of Tomorrow

The ideal college is Mark Hopkins on one end of a log and a student on the other. James Garfield

Every teacher worth their salt has learned the great secret of the profession. **They learned their discipline when they taught it**. The old saw is true, "to teach is to learn" or *docendo discimus* "by teaching we learn" a proverb attributed to Seneca the Younger[105] nearly 2,000 years ago that my great friend Frank Ferguson has taught me. I certainly found it so when I taught and finally learned the physics that I had not understood despite a myriad of college and graduate courses. For in order to teach a concept well, we have to understand it, link it to other ideas to place it in context, and be able to see it and show it in real world examples and exemplars.

Schools today make a sharp distinction between teachers and student. The distinction starts with the ceremonies and college degrees which have marked for nearly a thousand years the elevation of student to teacher. The graduation ceremony, the cap and gown, the pomp and circumstance, the parade, the diploma, all medieval rituals designed to define that transition, indeed the transformation that accepts former students into the ranks of those who are now distinguished as teachers. As teachers we are then told we have, all the "rights and privileges" to pass the knowledge and skills we gained at that institution of higher learning down to others. We will be measured, and we should measure ourselves by the fidelity our students exhibit as they present those knowledge and skills back to us.

In schools designed for the digital age, the distinctions we have for so long made between teachers and students fade away. Teachers are no longer primary sources of knowledge. Teachers may no longer be more skilled at using digital tools than their students. Teachers will no longer constantly play the central role in the front of the classroom forcing unity and maintaining central control and common order. If student-centered learning is, as we have seen, critical to classrooms in the digital age, and if we really believe that students learn by teaching, then students will have to teach. And who will they teach, well certainly each other, but they will teach their teacher as well. If they can ask "What if…" and create a new idea, if they can then

[105] In an epistle to his friend Lucretius.

teach their teachers that idea, then they will have not only learned that idea, that subject; they will have proven they learned it.

In our schools of the future, we will ask of this famous sculpture of Mark Hopkins and a student on a log, which is the teacher, and which is the student, for our students will be teachers and our teachers will be students, and we will reverse proverb, “to learn is to teach”.[106]

The Student of Tomorrow

“What do you want to be when you grow up?” I couldn't help but ask. Ariella was finishing her last year at UMASS in Amherst and volunteering at the WordPress conference in Boston called *WordCamp*. We got to talking during the long lunch break, “A user experience designer,” she replied. In earlier times, I would have thought such a bright, outgoing, vivacious young woman would have been in sales and marketing, perhaps retail, or even medicine. “Are you majoring in that at UMASS?” I asked. “Well,” she said, “They don’t really have a program in it, so I’m cobbling one together. Started out as a psychology major with a neuroscience track, dropped the track for just a B.S. in Psychology, added and dropped Communication as a secondary major, and have settled on a Psychology major with a minor in IT. The Communication department was not what I expected it to be and wasn’t offering the classes I was interested in, even though they were in the course catalogue. I think Psych + IT will give me a decent foundation for UX design since UMass doesn’t offer UX courses.”

She was planning on applying to a Web Development and User Experience Design accelerated program in Cape Town, South Africa for the next summer called iXperience. She also wanted to take some business courses she thinks she will need because she imagines working mainly as a consultant and running her own business. But the UMASS College of Business requires her to apply as a business major to be allowed to take any of their courses. I was not surprised. Most universities are still trying to prepare students for jobs that either no longer exist or jobs students are no longer especially interested in. Our kids are being forced to work along the seams, to cobble together majors to create their own vision of a career. The higher ed bureaucratic roadblocks they face at every turn can be frustrating and all too often deterring.

John Maeda, one of the foremost designers in the world and former president of the Rhode Island School of Design, keynoted the conference. When I talked to him after his keynote, he told me “**Students should be thought of as consumers.** They are our customers and we should treat them as customers; we, in PK-12 and higher ed are a business but we don’t act like

[106] I thank Peter Mili for this insight.

one." That insight was reason enough for me to have gone to learn more about how the WordPress community thinks about what it does.

WordPress is the development platform for a quarter of all the websites in the world. Ask Google how many websites there are in 2017 and it spits back 644 million. A fourth of that number would be north of 150 million websites built on WordPress. It is free. It is a community. It is constantly evolving. It drew a couple of hundred people to this conference just from the Boston area, to share ideas, experiences, and feelings. The conversations I participated in or eavesdropped on always included expressions from participants about community and the power of being a part of a creative shared enterprise. I could not help but feel they represented the future of education. They are our customers as well as our products. And as Maeda preached, inclusion must be central to our educational mission. For like Ariella, everyone in the community was making their own education, designing their own careers, and so loving being part of a community they volunteered to sit inside the Boston University Student Center on a beautiful summer weekend talking to people they had never met and likely would never meet again. I found it an exhilarating experience. I, too, gave up a beautiful day to be included in that future, for Ariella is the future of education. I, too, want our schools to listen to the workplace and prepare us for the jobs of the future. And I too, want our kids to be able to make their own future.

I was there to try to better understand why great websites are post-sites. When we first think about making a website, we imagine a series of pages that look very much like PowerPoint slides, often actually starting on PowerPoint. We add some links and sometimes a little animation and we think we are done. We are not. For if you do this in WordPress, you will soon find that there is a wealth of other things that you really need to attend to. There are these things called Posts that you use for doing a blog which constantly updates your site, for it is then immediately no longer static and Google loves it much more.

Posts are powerful things. If you are a Facebook, Pinterest, Snapchat user you are always doing posts, adding images, making announcements, building a timeline. For the idea of posting is not just to blog, not just a new form of communication, tossing out thoughts long or short on a daily or weekly basis. It is to grow an idea, to get others to comment on it, and elaborate it. In *What if Math* we use posting to add new Labs to our library for students to use. Posts can have categories and tags, keywords and hashtags, they can be sorted, galleried, or displayed as lists. They can contain images as well as text, videos as well as links, even tools to show them in a calendar or a timeline. And they can include comments. I think posts are the things that make WordPress a great development platform, a brilliant invention of this amazing age, and a tool for education in the digital age.

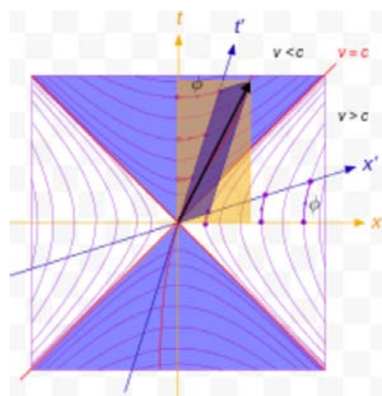
Unifying Curriculum and Instruction

"The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."[107]

Of all the beautiful passages dotting the landscape of physics this one by the mathematician Hermann Minkowski ranks at the very top. Minkowski was one of Einstein's teachers at the Swiss Polytechnic. Neither he nor his prize student found the other particularly stimulating or valuable, yet Minkowski was among the first scholars to recognize the revolutionary nature of Einstein's Special Relativity and the first to re-envision it.

As a physicist, Minkowski's words touched the very core of my scientific belief system. Space and time, the most fundamental of our concepts, defined iconically in the first pages of Newton's Principia and found in the first chapter of every intro physics textbook are called into question. Newton made them the axioms of Natural Philosophy, Einstein made them relative, Minkowski made them no longer basic, no longer foundational.

I cannot help but feel today that these same views apply to curriculum and instruction, to what we have long, long believed to be the core, the very foundation stones of education. In the digital age, as we have seen, we can no longer distinguish between teaching and learning, between what we view as curriculum and what we view as instruction. In Minkowski's elegant words, "only a kind of union of the two will preserve an independent reality." No longer can we talk about teaching and learning as two separate activities, separate tasks, by separate individuals. Lessons now incorporate all the aspects of curriculum, what we used to think of as the student's activity and of instruction as the teacher's activity. Lessons in the digital age are a "union" of the two.



An example of a Minkowski spacetime diagram

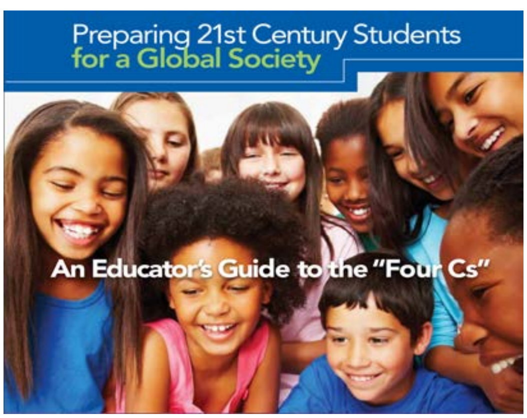
[107] Hermann Minkowski 1908

Our students will no longer separate them, going to one place for one and another place for the other. They will no longer go to lectures to get taught and go to labs or studios, or workbooks to practice and learn. Technology enables us to weave these two threads together to make whole cloth. This is not to suggest that 7-minute lectures are to be followed by student exercises in today's MOOCs is the future. I am sure they are not. For a union is a much more powerful idea making no fundamental distinction between these remnants of the old order.

In *What if Math,* our Labs are lessons, student activities which enable students to build their own models, try them, iterate them, and ask "What if…" of them. The models become the patterns to organize their experience, the content we associate today with lectures and textbooks, they learn by building those models. The iterative process and asking "What if…" replace the practice which today are separate assignments and worksheets. And the feedback, long a stable of instruction, is a natural part of building and iterating models. As students play with the parameter settings, they can watch the results in both the tables of values and the graph. Need help Google it, communicate with other students, or collaborate with others to make your learning more effective as well as more efficient.[108]

Which is now instruction, and which is now curriculum? We no longer can separate them or distinguish them. They are mere shadows and all we see is their union, the results of experiment.

Technology and the 4Cs



The 4Cs (Creativity, Critical Thinking, Collaboration, Communication) do not mention the word technology, but they were defined in the digital age and represent the digital world. The What if… question central to spreadsheets is a premier creative expression. "Out of the box" thinking questions have long animated Google and Apple's search for new employees. "Teams" are today common to creative corporations to solve novel problems because business believes in collaboration, in bringing together employees with diverse ideas, backgrounds, and interests to solve problems. Finally, the ability to communicate, long associated with just written and spoken language is today imagined in all its rich possibilities incorporating all 5 senses and more. Skill with the 4Cs is considered today to be the key to problem solving and success in work and play, even the NEA thinks so![109] Let's consider what these skills will look like in the digital age.

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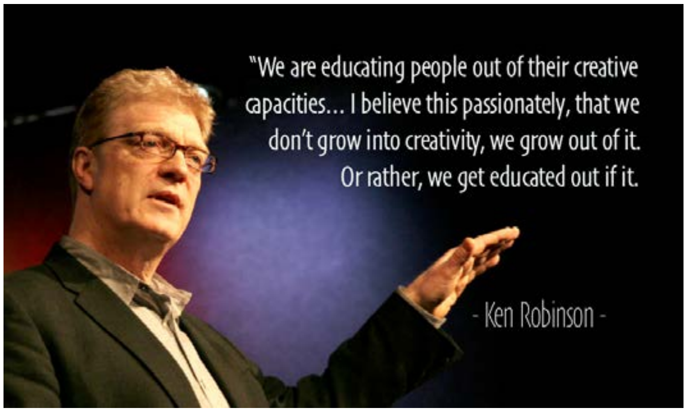
[108] In our iconic lab experience, high school chemistry, it is traditional to work in pairs on experiments.
[109] NEA http://www.nea.org/assets/docs/A-Guide-to-Four-Cs.pdf

Creativity



I imagine most of us would love to have a time machine not just to go backwards and bear witness to some significant event or to see a location we know looked like in a different age, but to go back in time to see our past selves at some interesting moments. One such moment occurred for me when I was in high school. I was taking a creativity test. Our school was part of a research project that I later came to believe was Jacob Getzels and Phil Jackson's work on creativity and intelligence.[110] Jackson and Getzels were later professors of mine during my master's program at the U of C, but sadly I do not remember engaging them in a dialog about the study, though I did read their book. I would thus love to go back in time and look at the tests, my answers, and how they categorized my creativity and intelligence. Would it make a difference? No! But it would be one of the more interesting time machine trips I could imagine, because we would all love to learn more about the origins of ideas, our own and others.

We have not come far in these past 60 years in either understanding or measuring creativity. We still rely, for example, on the "brick uses" test that I recall taking. "How many uses can you find for a common brick?" I seek not to enter the defining wars between creativity, novelty, imagination, out-of-the-box thinking. I do not wish to try to figure out how to measure it in some objective way which is likely not even possible. Nor do I believe the inborn creativity of young kids is smothered or destroyed by our rigid school curriculum and pedagogy. But I do think it valuable to establish some foundational notions that can guide us in pursuing education as a creative experience rather than to lay blame. For, I have long felt that creativity ought to be inherent in a 21st century education, and perhaps, just perhaps, its most valuable function.

I think we can all agree that creativity, whether we think of it as imagination, novelty, or out-of-the-box thinking, should be a primary skill we wish all our kids to have, learn, and use as a natural part of their school curriculum. **I think we can all agree that like every other skill, creativity improves with practice.** And I think we all agree that our kids and teachers should have many more creative opportunities and experiences in school. Once again, we have well-defined models for such schooling. Our arts education programs, particularly those oriented to design and design thinking, provide models.

With projects and problems that require performance, not single answers; with portfolios that collect students' work and enable them to demonstrate and display that work for others to view; and with juried exhibitions that are critiques and measures of those performances; schools of art

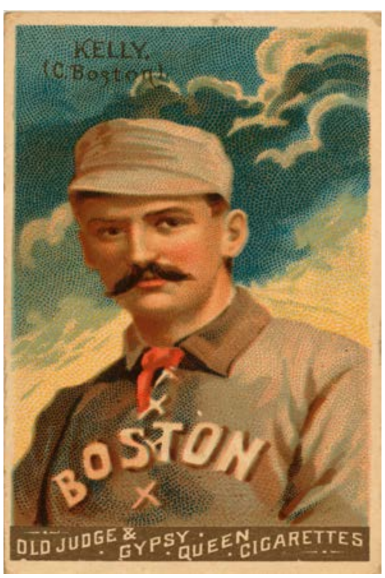
[110] Getzels and Jackson, *Creativity and Intelligence*, 1962.

and design can teach us how to bring creativity to all our subjects. These models open the door to new ways of measuring creativity and engaging students in practicing it. Across our student-focused educational vision, attitudinal surveys along with a variety of analytic metrics will provide teachers and students "measures of learning" increased interest, confidence, engagement, and creative expression.

In our *What if Math* lessons, Peter and I consider the most important part of the lesson the "What if…" question that we end every Lab with. Spreadsheets, as I have argued, are the "What if…" engines of our digital age, the creative question we want every model builder to ask. We believe that students who are learning problem-solving should be faced with this same question on every problem they work on, every model they build. **We believe creativity should be inherent in every learning experience.**

That we hunger for education to be a creative experience is borne out by Sir Ken Robinson's Ted talk in 2006, *How Schools Kill Creativity* with an astonishing 50,000,000+ views. There can be no more important thing we require in this reinvention of education then that it be a creative experience, for learning itself is an act of creation. While I do not agree with Robinson's basic premise that schools today destroy creativity, I do believe that they do little to stimulate or encourage it. Creativity like any skill is developed by practice and grows by stimulating practice. When learning is envisioned as a creative experience and becomes the experiences kids' practice, then their creativity will not wane, it will not be lacking, it will be their way of life.

Critical Thinking



The story of Billy Beane and Paul DePodesta who took a theory by Bill James and turned the Oakland Athletics baseball team from last place to World Series winner, was told by Michael Lewis in the book and movie "Moneyball", fascinating even to those who do not follow baseball. It is the story of a sport with a long tradition of using data and of compiling statistics. Like most of the baseball fanatic kids of my generation, I prized my baseball card collection with pictures of players on one side and stats on the other. These were precious treasures to be hoarded, traded, and displayed. I might guess that no other entity has been so data conscious for so long as baseball with the possible exception of the U.S. Census or the Stock Market. The metrics used to judge players and their performance followed a standard set of rules and stats that all clubs followed. Lewis tells the story of the general manager who broke the rules.

Lewis' story is valuable for educators because it gives us a peek into our future. All the baseball player metrics and statistics were and are proxies for something we cannot measure, their value to the team. How much does or could a player contribute to winning games? Value is very difficult to measure in any market, particularly sports. Who is more valuable to a team, the player with the most walks or the player with the most hits? Who will be more valuable to sign, the player who looks like our image of a great pitcher or the player who is overweight? Who is more valuable to have on your roster, the player who…? Baseball like every other business seeks to produce products of the greatest value. And schools do the same, we seek to make our class winners, the most desired, the highest scoring, the best contributors, the best ranking.

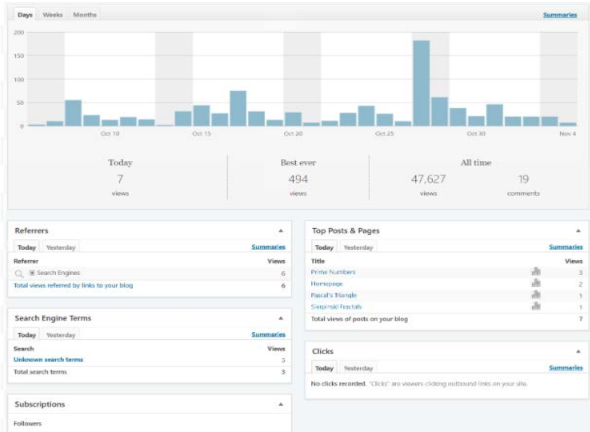
Billy Beane and his group changed baseball with two innovations. Their first was to focus on independent variables rather than dependent variables. Before them, baseball focused on stats like RBI (Runs Batted In). Beane focused on stats like walks and home runs. Independent variables are those that the player controlled independently of the rest of the team instead of stats that depended upon the situation on the field. He was able to measure and value the individual player without confounding factors like position in the batting order or quality of the team. Their second innovation was to look at players without the usual glamour commonly associated with scouting reports. They sought not "star" quality but growth and contribution. Walks may be boring, but they still put a player on base and increase the pitch count, so they are often more valuable than singles.

If we apply these innovation lessons to the schools of the future, we will want to measure students independently. Today, we generally measure students against their peers, grade them on a curve, critique them with our expectations for the class as a whole, score them on standard scales in tests. We do not grade students by their own performance or progress even though many teachers and schools may claim they are trying to do just that. We have sought to measure the value of learning each student and teacher produces. We give grades, test scores, and recommendations that are proxies for the amount and the potential for learning our students gain in our classroom, for we cannot measure learning directly. We cannot know what is happening in their brains. We cannot know how they will use what they did and did not learn in our classrooms. We cannot know the value of the year they spend with us for their future.

Like baseball General Managers, we too often as teachers, judge students on a narrow "Are they a good athlete spectrum?" We have a mental model of what a good student in our subject looks like, one who performs to our expectations. Are they good readers? Are they paying attention all the time? Do they know the answers to my questions? But like Billy Beane we have, in this new digital age, to judge our students across a much wider range of skills and abilities. Our communication technology is much more varied and richer. Our tools are far more powerful and malleable. Our roles much more flexible. Our students need not fit our expected pattern to be successful, to thrive, and to support the "team."

Evaluation or valuing becomes even more difficult when we tie it to learning "higher order thinking skills" like problem solving and critical thinking. We have little, if any, ability to predict how our kids will develop or use those skills in their homes and families much less in their schools or workplaces, but if we encourage them to find their own independent variables their measures of learning processes; then we and they will have succeeded beyond our highest expectations.

Collaboration and Communication



WordPress analytics for What if Math

Though the word goes back millennia, I had never heard it used until the last decade. Analytics, as Wikipedia defines it is, “the discovery, interpretation, and communication of meaningful patterns in data.”[111] The capability and the tools to do continuous analysis on data, big data, is barely a decade old in 2019. My son Arran is an analytics guy, a job category that did not exist at the turn of this century. His job is to help companies see and understand what their online presence is doing. He works with novel ideas like SEO[112] and with databases so large that powerful laptops take hours for Excel to crunch the numbers. The term and the idea have spread from those at the forefront of technology like Google to the conservative traditional school publishers like Pearson.

It is easy for educators and parents, particularly those who rhyme analytics with mathematics, to believe it a mysterious concept that further mechanizes education separating students from teachers leading to that awful vision of “machine learning” in which students, likewise, get plugged in. It is easy to become a Luddite, to fear that big data will take over our lives and turn our kids into robots. It is easy to view technology as negative gravity, a force pushing kids apart and pushing teachers and their students away from each other. I view it as just the opposite.

In so many ways technology and social networking has been a positive gravity. Today, they enable widespread sharing, working together on common problems, and even meeting up to further face-to-face interaction. Analytics can play a key role in education if we treat analytics as tools to help us understand what and when we are communicating, when our students are visiting our website, connecting to our links, downloading our content, answering our feedback. They are the tools that tell us what students find interesting and what they spend their time looking at. They tell us whether the information we thought important, we sought to communicate, agrees with their vision. And analytics are tools that help us understand how we can collaborate with our kids, and how they can learn to collaborate with others by linking people looking for similar things.

Though far from perfected today, analytics point the way to using technology to enhance educational communication and collaboration not only between teachers and students, and students and students, but teachers and teachers as well as teachers and educators creating new curriculum and instruction. Analytics today show the promise of mixing and enhancing

111 Wikipedia “analytics”
112 Search Engine Optimization

online and offline communicative and collaborative learning. And they provide evaluation tools for both students and teachers to grow and improve their collaboration and communication. These tools, today used for behind the scenes website management, could easily be opened to the students and the teachers themselves.

Analytics tools can enable students to see how well they are communicating with other students, which of their ideas, their posts, their blogs, and yes, their questions are shared. Analytics are already in the hands of our students. They know how to use them to: invite their friends, signal them with thumbs, or become followers. Our kids, indeed, all of us, have become addicted to these tools on social media, and yet we do not think of them as valuable for our schools or as learning tools, collaboration and communication tools for our students. They belong in our schools and our students need to learn to use them for problem solving. As I write this in 2019, Facebook is a 14-year-old student, Pinterest is 9, Instagram is 8, and Snapchat is just 7. It takes little imagination for us to appreciate a future when analytics tools like these, or ones to be invented, help our students collaborate and communicate in their digital age schools as they will in their digital age workplace, perhaps in a much richer fashion then even quality discussions face-to-face classrooms enable today.

Promoting Choice

“Because we're going to sell them cheap books and legal addictive stimulants…”[113]

I can still picture the street vendors I saw in Seattle in the early 1980’s selling flavored coffees to tourists, workers, and just plain passersby, and wondering what the fuss was all about since I had not even tasted coffee for over a decade before that. I found them fascinating and irrelevant, but Howard Schultz did not. The story of Starbucks is well known. He coupled varied coffee tastes with the Italian coffee house atmosphere to bring something new to America. While I still don’t drink coffee, I love coffee houses and often find a comfortable spot in one to spend part of my day working, writing, and talking.

Today, libraries have joined books with the coffee house experience. College libraries and even public libraries have become coffee houses as well as book lending repositories, delightful

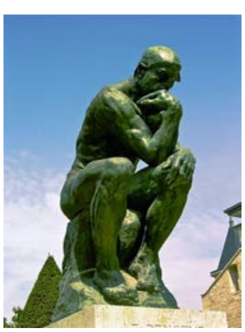
[113] *You’ve Got Mail,* written and directed by Nora Ephron, 1998

places to spend an afternoon thinking, working on a laptop, interacting with classmates, or just talking with friends. The libraries of the future are places where legal addictive stimulants like coffee or tea, the Web, and collaborations are joined together. They are no longer just shelf upon shelf, row after row of spines stacked and ordered by the 19th century Dewey Decimal System searchable by card catalogs filling hundreds of small drawers. They no longer have silence signs and individual reading lamps. They are active and interactive classrooms with comfortable nooks and crannies along with desks and upright chairs.

In this digital age, libraries like schools, must serve a different purpose. They are not just places we march off to in search of a reference or a book. They are places we go to be “sold” to overcome our natural inertia and start a new thing. A book, a magazine, or a DVD, like any new learning experience, must be sold. We have to want to spend the time on it, and usually do that because a specific need, an attention-grabbing cover, or a friend’s or critic’s recommendation. So today, the libraries of the future turn their books to face out on shelves, to show their cover. They put books on a themed display, post staff picks to grab the passerby’s attention, and sponsor author readings so common today. Like Amazon, they suggest and “sell” books by mining your data and engaging you in social media experiences. They recognize that in today’s overcrowded world, attention is paramount, and everything is a sale.

For libraries to remain relevant in the digital age, they must sell legal stimulants. The same goes for our schools and for our content. The old paper syllabus that simply listed topics, now copied by an ugly flat text-based interface, is not designed to stimulate. It is not a store that “sells” learning. It does not compete for a student’s attention or treat students as customers. If we want our classrooms to be relevant, if we want to entice students who have choices, if we want to engage our students, then we must remember to provide those stimulants. Like a modern bookstore we must give our customers choices and experiences. We constantly have to try to “sell” them, to capture their imaginations not just with bright and even animated graphics like the iTunes or app stores, but with stories they could find fascinating. They do not have to be lured with a caffeinated drink, but they do have to make our kids want to click on them, work on them, share them. We have to remember that our students are our customers, and they are motivated by what is most relevant to them. We have to show off our content to entice students to it and make learning addictive for every student.

Promoting Concentration



We see the results in sports all the time when a player or a team loses concentration: the infielder lets a ball go through his legs, the quarterback throws an interception, the basketball player makes an errant pass, the golfer sends the ball into the rough. Concentration may well be one of our most precious “fluids”. When we tire it flows away. Use it up and it takes us a while to replenish it. Waste it and we have to wait to replenish it. We may not know how we lose it, but we do know for sure that we do not learn without it. Long ago, the dream of sleeping on a pillow with a speaker under our ear to teach a foreign language has been shown to only disturb sleep. For learning takes concentration, alert, directed, focused concentration.

More recently the computer/communicator we all now carry in our pockets has many of our kids spouting a new concentration fiction – multitasking – believing they can text and carry on a verbal conversation at the same time, believing they can drive and text, and they even believe they can learn and Facebook at the same time. They believe that their generation, steeped in digital technology, can actually split their concentration and learn even complex problem solving while playing a video game.

I am not here as an old fogey to try to dissuade them of this fiction, for I can certainly not do that. But I just hope my back surgeon is not multitasking in the middle of my operation. I have spent my life concentrating on hard ideas, and I am deeply fortunate to have, what I consider, very good concentration. I can work in coffee shops without plugging my ears with music to shut out the din of background conversation. I can work on ideas and tasks for long periods of time and stay with an idea for longer than most. Some would call it patience, I believe it is much more valuable than that. I know when I have it. I protect it. I husband it. And I know when I don't. Even when I am desperate, highly motivated to finish a project or write as little as one more page and can't concentrate, I cannot do it or at the very least to it well. It is not a matter of practice, desire, or grit. When you are out of concentration, you better take a break.

> *"We have to stop seeing boredom as a frilly side effect. It is a central issue. Engagement is a precondition for learning," he adds. "No learning happens until students agree to become engaged with the material."* Jal Mehta, Harvard Graduate School of Education

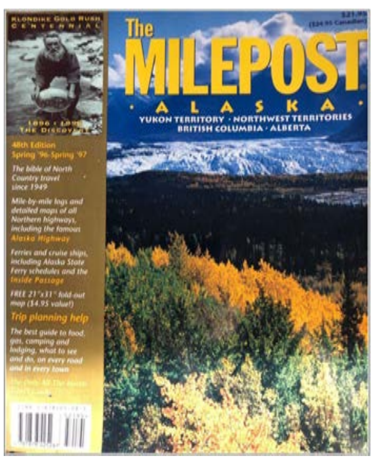
Of course, I can walk and chew gum at the same time. I can drive and talk with someone else in the car at the same time, and I can work and listen to music at the same time, but these tasks obviously use different parts of our brains, so they do not have to share concentration. Their very automaticity enables me to concentrate on one in common situations and do the other. But if I am driving through blizzard like conditions, I cannot and do not talk about big ideas. If I am trying to analyze the music I am listening to, then I cannot also be trying to write an introductory paragraph to an important paper. If I am trying to solve a significant problem, I can't be making an important life decision at the same time. Perhaps, in some situations, each of us can share concentration, but not when we are trying to learn, for learning anything, especially anything new, takes real concentration.

For most of us, concentration is not under our conscious control. How often have you said to yourself, "concentrate now" without any result? Our conscious mind does not have a switch to turn concentration on and off. If we do not have confidence that we can learn something, we find it hard to concentrate and often cannot ever do it. If we are bored with something, the concentration switch turns off and remains out of our control. If we find something unimportant, concentration goes off by itself and our mind wanders. This connection between learning, concentration, confidence, and caring, seem to me to be fundamental human characteristics. They play out in our schools every day.

Students who find a subject uninteresting do not concentrate on it and do not learn it. Students who believe they cannot learn a subject do not learn the subject. They never concentrate enough on its ideas to understand or use them. Students who do not believe a subject is important, who do not give it more than cursory concentration, do not retain it. As teachers, we think of those students as poor learners, undisciplined, as lacking in motivation. We often think of them as not being smart. We spend hour after hour in our classrooms trying to persuade and motivate them, exhausting ourselves in the process. We try to put those “poor” students into remedial situations, slowing down the learning process to make it even more boring. Or we do the “I believe in you” dance, to try to convince them that underneath they have the smarts and can do the learning. In some magical cases, teachers may be able to make this strategy work,[114] but despite having known great wonderful teachers who do magical things, I rarely, if ever, have seen magic concentration stories replicate or scale.

As long as schools focus on the same lessons for all, we have not and will not solve this concentration problem. It is not up to the teacher to solve it. It is entirely up to the student. And for students to solve their own concentration problem, they have to care about what they are learning, believe they can learn it, and husband their concentration for that purpose.

Promoting Assets



I took the road less traveled to Alaska one summer not so much for adventure but for discovery. It was my lifetime sabbatical dream to drive the Trans Alaska Highway, as some call it, to see and feel the kind of landscape that may have been the path the first human settlers took after they crossed Beringia from Asia to inhabit a new continent. I drove the length of the highway from the great Canadian national parks of the Rockies to Anchorage with two of my kids. It turned out to be both memorable and adventurous after all. I tell you this story because of a book, *The Milepost*, the definitive guide to the highway, and a metaphor for much that is wrong with our education today.

The 1500-mile highway was built as an emergency measure in the early days of World War II to supply soldiers in the territory of Alaska in case of a Japanese invasion. It was originally a gravel rough road. When we travelled it, it was a two-lane highway, usually paved, where the phenomenal scenery was very rarely interrupted by any other vehicles. When the highway was constructed, mileposts were placed as markers and soon after the war the book came out to help adventurers drive the road. Filled with ads for places to eat

[114] The Jaime Escalante AP Calculus story has become the iconic example.

and stay, as well as stories/suggestions for places to visit, it was our Bible for the trip. Sometimes it protected us by finding places we could fill up, eat, and sleep. Sometimes it led us astray suggesting a “modern” motel where I would not let my kids turn on the propane stove in near freezing weather for fear of carbon monoxide poisoning, when a brand-new beautiful lodge was available just 10 miles further down the road.

Much of our curriculum, especially in mathematics, is akin to *The Milepost*. It is a long linear path with a myriad of stopping points marked along the way, each with its activities (adventures) and concepts (stories) that we cannot miss. Our curriculum is today a progression, just as our trip was, from one milepost to another. Miss a milepost and it is hard to go back, driven by the need to meet the tight schedule in relentless pursuit of “covering the material” that marks our schools today. Miss a milepost in our courses and far too often, we create a difficult to remedy deficit. Unlike a highway on a vacation trip where we can choose the mileposts we want to explore further and enjoy our side trips where little is dependent on our choices, our schools are linear, driving toward competencies which we believe build one on another.

Our milepost curriculum makes our classrooms deficit-based, where we keep trying to fill in the holes as we keep moving students forward; where we worry about what kids don’t know and not what they do know, where we focus on their deficits and not their assets. As we reinvent education for the digital age, we must turn our classrooms from deficit-based to asset-based[115] no longer looking at a student’s deficiencies but focusing on their strengths.

Technology

New Apple iPad

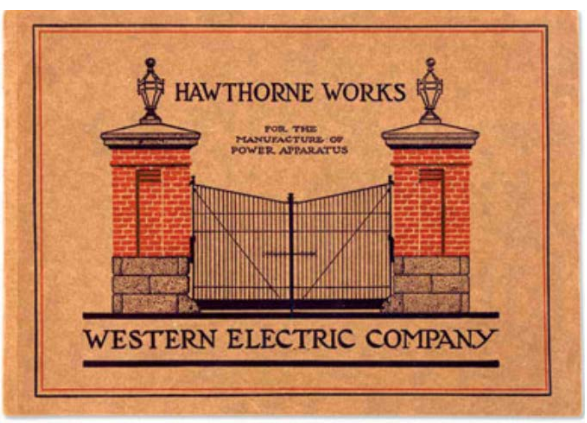
When my friend Steve Bayle, ed tech pioneer, hard edged thinker, rare to praise, and in recent years highly critical of Apple, sent me an email after watching the latest Apple event in Brooklyn he ended with one word “**Mind-blowing!”** I knew things had changed. Apple was showing off their new iPad, now more powerful than most laptops, gorgeous screens, interactive pencils to draw and edit that magnetically attach to the tablet just as the tablet attaches to the keyboard. It seemed to both of us that this product, which had been losing sales and looked to be dormant for a couple of years, had just matured, becoming a computer that perhaps these two business folk wanted as their primary computer. Mind-blowing is right!

[115] I am grateful to Peter Mili for this insight.

The history of technology is filled with revolutionary technologies which follow a comparable and inevitable cycle: first automate the old technology, then iterate that new technology to substantially improve performance, and finally, profoundly, reenvision the technology, change, the paradigm, and revolutionize our world. Technology is capable of that. Electric lights coupled with electric motors made us self-sufficient enabling the middle class. Automobiles and expressways remade our landscape by moving us to suburbs. And cheap air conditioning, improved rapid transit, and clean air technology moved us back into cities. In each case, technology solved a problem technology had created. In each case, technology integrated processes we long thought distinct. In each case technology made it possible for us to do things we had not imagined we would do.

The same will hold true for education. Our schools, overwhelmed by the demands technology has placed on them and on society, are about to be transformed by technology. And we can expect, as with all such technology induced changes, that this one too will make the products and services we build and consume more relevant to our customer, more effective allowing our customers to do much more, and more efficient to cost us and them much less. Did this new iPad integrate our smart phones with our computers? Does it enable us to have a single interface, for all intents and purposes a single device, I cannot say? But whether it does or does not, our educational digital technology is in my view rapidly approaching that third level ready to change our world, and make schools relevant, effective and efficient for all our kids.

Relevant



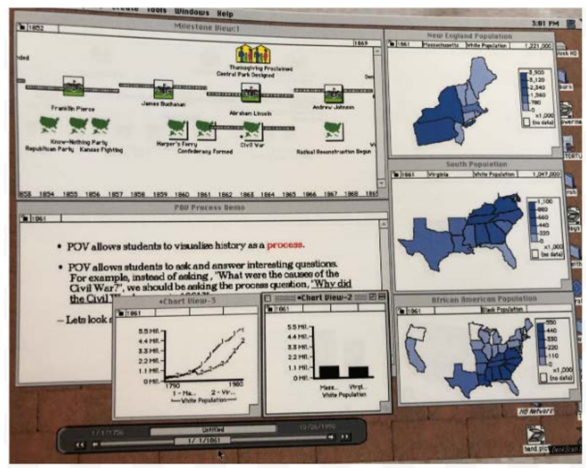
One of the most famous case studies

It was introduced in the 1920's as a new teaching methodology along with the new Harvard Business School MBA. The Case Study has since spread to most business schools and professional schools around the world. Today, HBS has over 30,000 case studies in its vaults and students study over 2,000 in their pursuit of an MBA. New ones are added, old ones are mothballed keeping the library fresh and relevant. While they may differ in context in different courses, they are common in form, and their essential purpose is always to develop in HBS students a wide-ranging problem-solving capability. For the most part, HBS education is not about building business knowledge but about building business problem solving skills. The faculty and students know that while knowledge in the business world is constantly changing, the skills to solve problems and to obtain new knowledge remain constant.

I was first introduced to case studies in the mid-1980's. As an educational software pioneer, I was often invited to meet with people wanting to rethink their businesses in response to the then new personal computer. I remember one meeting in particular because of its office. I walked into a large open lobby with a half dozen or so perimeter offices on the two sides of a right triangle with big windows for executives to enjoy. On the other side of the lobby, behind the hypotenuse wall, there was a huge bank of matched 5 drawer lateral file cabinets and the biggest Xerox copy machine I had ever seen spread across the center. I was told that this business was minting money selling case studies to business school students.

They had the process down to a science. Each semester they would send out a catalog of cases to professors who would check off the cases they wanted to use in their classes and the number of students expected in each class and send it back. A technician would take the sheet, pull the file folders of the checked case studies out of the lateral drawers, load them into the gigantic copy machine on at a time, key in the number of copies and out came a pile of collated, stapled booklets ready for shipping. Add a few new titles each semester, send out a new catalog, and wait for the bookstores to send you a check. They set a standard price, 3 cents per page with an all-in cost about half that. Great business, I salivated, I would love to have an automatic renewable low overhead business.

A fully optimized paper technology business, Gutenberg would have found it an amazing evolution of his printing press. Print on demand, up-to-date, with flexible choices to meet the needs of most customers (in this case professors). But as sweet as it was and perhaps still is, now augmented by pdf technology, this is still a paper-based business in a digital world. It is also a whole class business driven by teachers with standard and common problems for every student.



Point of View 1988 from Learningways

Not long after I saw this business model, I got the chance to reinvent it for the digital age, to work on an HBS digital case study using our amazing *Point of View* technology. Teamed with a professor at HBS, a brilliant designer Bryant Patten and with an amazing research and analysis group Oxford Analytica in England. *Point of View*, one of the early Macintosh programs, tied together databases, timelines, maps, visuals, and text to produce a truly interactive way to visualize databases. For this particular digital case study, the databases dealt with international economics. The result was extraordinary and novel, but it did not fit that HBS case study business model.

The paper business model split the revenue between the Business School, the professor, and the printer, with spare change going to the grad students who collected the data. The digital business model had to also include our company Learningways, Oxford Analytica, and a team of digital content producers who found and incorporated the graphs, text, maps, and other visuals. Our view of this amazing interactive, visual product no longer fit the model of an automatic printing press. Production costs were now in the 10's of thousands and no longer in the hundreds of dollars. I would no longer invest in this business.

The cost of development and cost of sales broadly speaking were not the only problem with this technology solution. It also failed to be a model of the future case study because it was no longer based easily made relevant. It was a big, general case study, a large-scale project-based learning lesson that could not be easily modified or cloned to make technology-based thousands of case studies. It was not a model we could imagine in its rich content form that could be made relevant to the desires of the professor or interests or dreams of the student. New models, both of business and of content are here needed[116] if students as well as teachers are to have choices. While spreadsheets lack some of the timeline/dynamic capacity of *Point of View*, I think they enable us to ask the display and "What if…" questions *Point of View* pioneered. They have the potential for individual prolific development to make them cheap and relevant. As we think about technology, we must imagine flexible real-world platforms that can enable large numbers of people to participate in the development of generally cheap and varied lessons that are easily kept or made relevant, for they are essential to real choice.

Effective


Leibniz Stepped Reckoner, 1673 Calculator to add, substract, multiply, divide

Ask 5th graders to name the hardest idea in math, and they, like most adults, will say calculus. Ask high school students what AP test they must pass to get into a very good college, and they will say calculus. Ask college graduates what subject they never really understood, and they will say calculus. Its name, today a synonym for subjects that are demanding to learn, comes from the Latin for small stones, like those used in a Roman abacus. Much of its complexity is well deserved, for we are taught calculus today as an analog subject dealing mainly with continuous functions, spending significant time and effort on hard abstract concepts like limit, continuity, infinity, infinitesimals, slope of a curve, tangent line to a curve, and delta/epsilon proofs. These ideas are not only difficult to understand, they are taught at the introduction to the subject. This approach might have made eminent sense in an analog world where most functions are continuous, where sophisticated algorithms were necessary for finding solutions, and where discrete functions require extensive exhausting calculations. But they made its central concepts numbingly abstract.

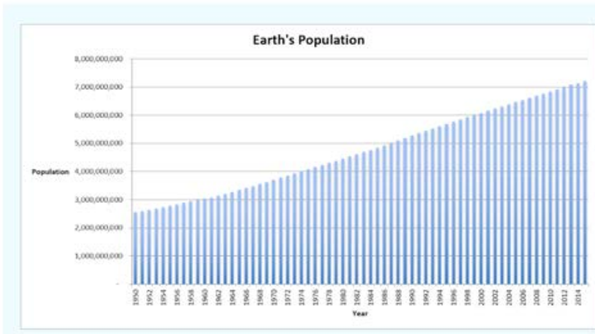
I have to admit I flunked calculus in college. Perhaps that's a reason I am so determined to use technology to make calculus much easier to understand. I believe it is important for digital age

[116] See my book *New Physical Ideas are Here Needed*, Art Bardige, 2007

problem solving for all our kids. I believe digital technologies especially spreadsheets can enable all our kids to learn calculus effectively. And I believe it will just plain make our kids feel and be smarter.

The derivative is a ratio, a rate, the rate of change. Digital technology lets us treat functions as discrete and thus to use simple arithmetic to find values by taking the difference or the ratio between each of its values. The integral is the total change, a sum and not a difference, a product and not a ratio, and it too can be found digitally by using simple arithmetic just the sum of its values of the function. We are adding and subtracting.

Year	Population	Growth
1950	2,556,000,000	
1951	2,594,000,000	
1952	2,636,000,000	
1953	2,681,000,000	
1954	2,729,000,000	
1955	2,780,000,000	
1956	2,833,000,000	
1957	2,889,000,000	
1958	2,945,000,000	
1959	2,997,000,000	
1960	3,039,000,000	
1961	3,080,000,000	
1962	3,136,000,000	
1963	3,205,000,000	
1964	3,276,000,000	



We can ask students to decide for themselves whether the rate of growth of the world's population is speeding up or slowing down by taking the real data and finding and plotting the differences between each year of the data. By doing that they are computing the slope of the "curve", they are differentiating. And if we ask them to forecast the earth's population 25 years from now, they can take that rate of change and us it to continue the curve by summing its values, by integrating. They can do the same with GDP, once again with real data, once again finding the rate of change and predicting GDP growth. These images are taken from our real *What if Math* spreadsheets. Students in 5th grade will be using the fundamental ideas of calculus to ask "What if…" about real world problems.

Year	GDP in billions	Population in millions	GDP per Capita	Inflation Rate	GDP Constant $	Rate of GDP Growth
1929	104.6	121		-1.16%		
1930	92.2	123		0.00%		
1931	77.4	124		-7.02%		
1932	59.5	124		-10.06%		
1933	57.2	125		-9.79%		
1934	66.8	126		2.33%		
1935	74.3	127		3.03%		
1936	84.9	128		1.47%		
1937	93.0	128		2.17%		
1938	87.4	129		0.71%		
1939	93.5	130		-1.41%		
1940	102.9	132		-0.71%		
1941	129.4	133		1.44%		
1942	166.0	134		11.35%		

Jim Kaput was right.[117] These fundamental and important ideas can be learned by 5th graders. They can learn function-based algebra and calculus both critical concepts in our digital age by using spreadsheets to work with discrete quantities. Technology can make hard ideas easy, producing effective learning for every student at earlier grades, making students smarter.[118]

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[117] Kaput developed lessons in calculus for inner city 5th graders and proved they could understand its fundamental concepts.

[118] The calculus of Newton and Leibniz focused on continuous functions and the algorithms for finding the rate of change and total change, the derivative and integral of those continuous functions. Their general simplicity reflects the power of mathematics to use simple, elegant patterns. I am not advocating that this

Efficient

In 1960, the K-12 student-teacher ratio was around 26 to 1. Today, it hovers near 15 to 1[119]. During the past half century education has believed "less is more" and "small is beautiful". Smaller classes have been the mark of "good schools". Special Education, focused on Individual Education Plans, has featured very small student-teacher ratios as standard requirements for a quarter of our kids or more. This race to effectiveness by reducing efficiency is unsustainable, a broken business model in both PK-12 and higher education. It has driven up the costs of education so that affluent suburbs thrive and schools in less affluent big and small cities and rural areas do not. It has made college unaffordable, driven student debt beyond their ability to repay it, and is making tight budgets and even survivability the question every private schools and colleges face.

If we are to become the nation our founders dreamed of, if we want the vast majority of our kids to thrive in the digital age they will live in, then we must double student college graduation rates. To do that without bankrupting them or our nation, we not only have to substantially increase the effectiveness of our schools, we must do it at half the cost. To meet such audacious goals, we obviously need a new business model, the old one is broken, it can no longer be tinkered with, no longer improved enough; It no longer works in the digital age.

The tools of the digital age with their great online bandwidth[120] and utility enable us to unify curriculum and instruction, connect students instantly to the free (for all intents and purposes) Web "library" of all human knowledge, and encourage students to collaborate and communicate with each other and with their teachers. These tools lead us to expect students to play a much larger role in their own education, to learn independently, responsively, and energetically. To taking full advantage of this power they will need to become consumers with access to most digital age tools, a rich range of choices of content, problems, and contexts, and the opportunity

beauty be taken out of the mathematics students learn, but rather that once they understand and can use the concepts of calculus with discrete functions, there is plenty of time and space to enable all our kids to see the wondrous algorithmic patterns in the calculus of continuous functions.

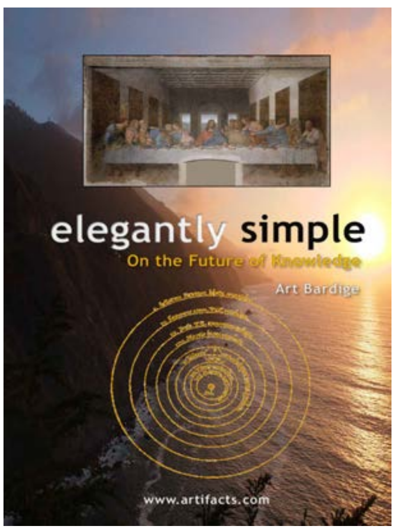
[119] In 1952 my 5th grade class in the city of Chicago had 51 students 6 columns by 8 rows plus 3 moveable desks, a 1 to 51 teacher-student ratio and no special education relief.

[120] Bandwidth enables us to use text, static images, dynamic images, video, sound, interactivity and even face-to-face or group conversations. Books give us only text and static images.

to be creative and share their creative work. These are the primary elements of a digital age education that seeks to "make it real".

Using technology to support and supply most if not all the direct instruction, getting or producing content in a problem-based learning format, and making careful and economical use of high-quality teacher time is at the heart of a new business model. Opening the Web to all students and all teachers, with minimal restriction, would enable them to learn how best to find what they need, take control of their own education, and set themselves on the path of lifelong learning. Finally, reimagining the skills and content we base our schools on, from the 3Rs to the 4Cs, aligns us with digital age reality, and the needs of business, turning students into teachers and teachers into students and imbuing both with the spirit of entrepreneurship.

Liberal Arts and The Invention of Knowledge



Without the tyranny of a demanding reading emphasis and a domineering overstuffed math curriculum, we can find time in both K-12 and higher ed for liberal arts and civics, two subjects woefully underrepresented today. Liberal arts have a long and valued tradition as suggested by this quote from Harvard University president Larry Bacow[121]. Its value has not diminished in this digital age for in the broadest sense, a liberal arts education is about learning to think, and if there is anything more important today than students learning to think and especially to think out-of-the-box, I don't know what it is. The skills they will need to think flexibly, continue learning, reinvent themselves lifelong, and the knowledge of the common threads that bind us as humans are all critical to our future.

> A broad liberal arts education--and its emphasis on critical thinking and rational argument--has never been more important, and our students grow intellectually in ways that will prepare them not only to seek truth and wisdom, but also to become sources of both.

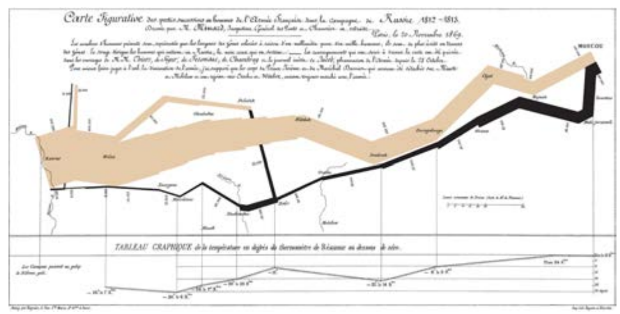
In my book, *Elegantly Simple,*[122] I found a common pattern to the history of knowledge and a common foundation to the construction of knowledge across all disciplines. Such a pattern fits with our vision of education, providing a tour uniting all the disciplines across time and space, and an opportunity to explore any ideas of interest and any creations in detail. "The Pattern of Knowledge[123]" included in that work could serve as a "Periodic Table of Knowledge" for

[121] *Harvard University Magazine*, January/February 2019
[122] Art Bardige, *Elegantly Simple*, www.artifacts.com
[123] The Pattern of Knowledge https://artifacts.com/wp-content/uploads/2014/01/Pattern-of-Knowledge-pok.complete.pdf

students, teachers, and lesson developers. Such a pattern replaces the linear structure of most liberal arts courses. Like the Periodic Table, it enables students to visualize the *elements of knowledge* across the disciplines, enabling students to study in depth any cases they choose or are given, while keeping in mind the connections between the various arts and sciences.

There is great power in having both a broad view of the history of ideas and detailed knowledges of a local kind. The problem we have long faced in education is that we have only given that power to students who have passed through many gates. All too often they must, at a minimum, have completed two years of college. The view of schooling I have laid out here lets students begin to learn to apply this great power of seeing the whole while working on the parts from the very beginning of education.

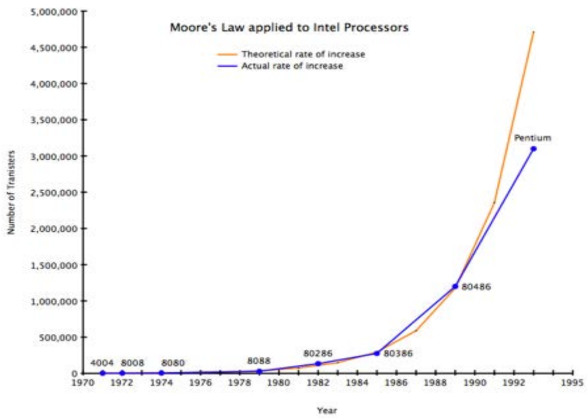
Of course, some may wonder whether the liberal arts can be learned without being a highly skilled reader. To that I ask you to consider whether Shakespeare's plays are best read or heard? For surely, they were designed for the stage. Poetry is to be read aloud as it is to be read silently, to be joined with music as much as read silently, to be shared with visual images as much as it is plain text. History found in a map can be the source rather than the enhancement. Psychology and economics need not be found in overblown textbooks, but instead in dynamic visual interactive experiments. The multimedia of our digital age enhances the liberal arts and enables students with strong skills other than reading text to enjoy and participate in them. Digital age technology can remove what we have long thought of as the primary skill barriers while making clear the powerful patterns so long our challenge.



		SYMBOLS (Earliest Use)		SYMBOLS (Medieval Europe)	UNIVERSALS (Greece/Rome)		UNIVERSALS (Medieval Europe)		OBJECTS		ENVIRONMENTS		ARTIFACTS
		Singular	Plural	Plural	Singular	Plural	Singular	Plural	Singular	Plural	Singular	Plural	Singular
		Tribal Societies	First Civilzations (Empires)	c. 800	c. 600	c. 440	c. 1050	c. 1250	1498	1686	1858	1900	Examples
PARTS	Connections	bone marking cave painting language tombs tools	arithmetic c.3000 bronze tools c.3000 calendars c.3000 cities c.3000 hieroglyphic writing c.3000 inventories c.3000 money c.3000 stone buildings c.3000 stone sculptures c.3000 ziggurats c.3000	*1001 Nights* c. 800 Aachen Chapel 792-805 Alcuin 735-804 bureaucracy c. 800 Carolingian script c. 800 Charlemagne r768-814 coinage c. 800 Holy Roman Empire c. 800	Aesop 619-564 Anaximander 611-547 Archaic Art c. 600 Black Figure Vase c. 600 *Deuteronomy* 638-609 Doric Order c. 600 Kouros c. 600 Lao-Tse 604-517 Polis c. 600 Solon c. 640 Thales 624-546 Zoroaster 660-583	Democritus 460-370 Dying Niobid 450-440 Euripedes 485-406 Hippias 460-? Hippocrates 460-377 Parthenon 448-432 Protagoras 480-411 Socrates 469-399 Thucydides 460-404	Bayeau Tapestry c. 1083 Church of St. Etienne c.1062 city-towns c.1050 College of Cardinals 1059 free-standing-statues c.1050 guilds c.1050 Pope Gregory 1073 Romanesque art c.1050 Romanesque c.1050	Roger Bacon 1214-1294 Albertus Magnus 1193-1280 Parliament 1268 Parzival c. 1250 St.Urban c. 1261-1275 Thomas 1225-1274	Bramante 1444-1514 Copernicus 1473-1543 DaVinci 1452-1519 Durer 1471-1525 Erasmus 1466-1536 Luther 1483-1553 Machiavelli 1469-1527 Michelangelo 1475-1564 More 1478-1535 Paracelsus 1493-1541 Raphael 1483-1520	Berkeley 1685-1753 Bernoulli 1654-1705 Defoe 1660-1731 Halley 1656-1742 Huygens 1629-1695 Leeuwenhoek 1632-1723 Leibniz 1646-1716 Locke 1632-1704 Newton 1642-1727 Swift 1667-1743 Watteau 1684-1721	Carroll 1832-1898 Cayley 1821-1895 Darwin 1809-1882 Dedekind 1831-1916 Dickens 1812-1870 Manet 1832-1883 Maxwell 1831-1879 Mendel 1822-1884 Pasteur 1822-1895 Rodin 1840-1917 Ruskin 1840-1917 Tolstoy 1828-1910 Victoria 1819-1901 Wagner 1813-1883	Bergson 1859-1941 Conrad 1857-1924 Curie 1867-1934 Dewey 1859-1952 Freud 1854-1939 Hilbert 1862-1943 Matisse 1869-1954 Pavlov 1849-1936 Planck 1858-1947 Poincare 1854-1912 Russell 1872-1970 Shaw 1856-1950 Wright 1869-1959	Guggenheim Bilbao 1997 *Outliers* 2008 Superstring Theory c. 1995 *The Black Swan* 2007 Unique Artifacts 1995
					c. 525	c. 400	c. 1075	c. 1300	c. 1540	c. 1740	1869	1905	
PARTS	Relations	kinship/clans totemism Venus Figures	Hammurabi c.1750 husband&wife c.2600 pyramids at Giza c.2500 symbolic cuneiform Zoser Step Pyramid c.2650	legal codes c. 800	Buddha 563-483 Confucius 551-479 Psiax Vases c. 525 Pythagoras 569-500	Archtas 428-347 Eudoxus 408-355 Lysippus fl. 370 Plato (early) 429-347 Theaeteus 415-369	Anselm 1033-1109 Crusades 1095-> Rashi 1040-1105 Scholasticism 1100-> troubadours c. 1100	Meister Eckhart c. 1260-1327 Pisano fl.1258-78	Calvin 1509-1553 P. Bruegel 1525-1569 Tartaglia 1500-1557 Tintorello 1518-1594 Trent Council 1543-1563 Versalius 1514-1564	Bach 1685-1750 Bernoulli 1700-1782 D'Alembert 1717-1783 Diderot 1713-1772 Euler 1707-1783 Fielding 1707-1754 Franklin 1706-1790 Hartley 1705-1790 Hume 1711-1776 Johnson 1709-1784 Linnaeus 1707-1778 Rousseau 1712-1778 Voltaire 1694-1778	Boltzman 1844-1906 Brahms 1833-1897 Degas 1834-1917 Mendeleev 1834-1907 Monet 1840-1929 Renoir 1841-1919 Twain 1835-1910 Whistler 1834-1903	Bohr 1885-1962 Eddington 1882-1944 Einstein 1879-1955 Jung 1875-1961 Kandinsky 1866-1944 Malinowski 1884-1942 Minkowski 1864-1909 Mondrian 1872-1944 Picasso 1881-1973 Rutherford 1871-1937 Santayana 1863-1952 Watson 1878-1958 Wright 1869-1959	
					c. 500	c. 370		c. 1310	c. 1580	c. 1775	c. 1875	1914	
PARTS	Transformations	initiation rights pottery	carved temples c.1500 Hatshepsut Temple c.1480		Dying Warrior c. 490 Heraclitus fl.500 Herakles c. 490 Kore (Chios?) c. 520	Mausoleum c. 359-351 Mausolus c. 359 Plato (late) 429-347 Temple of c. 360		Boccaccio 1313-1375 Dante 1265-1321 Giotto 1275-1337	Brahe 1546-1601 Cervantes 1547-1616 El Greco 1541-1601 Elizabeth 1533-1603 Gilbert 1540-1603 Montaigne 1533-1592 Napier 1550-1617 Shakespeare 1564-1616 Stevin 1548-1620 Vieta 1540-1603	Cavendish 1731-1810 Coulomb 1736-1806 David 1748-1828 Gibbon 1737-1894 Goethe 1749-1832 Goya 1748-1828 Haydn 1737-1809 Kant 1724-1804 Lagrange 1736-1813 Lavoisier 1743-1794 Mozart 1756-1801 Priestley 1733-1804 Smith 1723-1790	Klein 1849-1925 Rodin 1840-1917	Bauhaus 1918-1932 Buber 1878-1965 Einstein 1879-1955 Fitzgerald 1896-1940 Heidegger 1889-1976 Heisenberg 1901-1976 Joyce 1882-1941 Kafka 1883-1924 Le Courbusier 1886-1965 Piaget 1896-1980 Proust 1871-1929 Schroedinger 1887-1961	
		Tribal Societies	Empires		c. 480	c. 350	c. 1137	c. 1350	c. 1610	c. 1800	c. 1883	1927	
WHOLES	Connections	chiefdoms farming gods kings priests shamans towns tribal Europe	Akhenaten c. 1365 Assyrians great empires feudal society feudal manor c. 900	feudal Europe	Aeschylus 525-456 Classical Art c. 500 Parmenides fl. 500 Red Figure c. 520	Aristotle 384-322 Epicuras 342-270 Euclid 330-275 Library- c. 300 Praxiteles c. 390-330 Zeno the Stoic c. 330-?	Abbot Suger 1081-1151 Abelard 1079-1142 Age of Chivalry c. 1100 Church-of-St.-Denis 1140-1144 Geoffrey of c. 1100 -1155 Gothic Architecture c. 1137 Portrait of Physician c. 1160	Buridan c. 1300-1358 Chaucer 1343-1400 Ockham 1295-1347 Oresmi 1323-1382 Petrarch 1304-1374	Bacon 1561-1626 Caravagio 1573-1610 Donne 1573-1631 Galileo 1564-1642 Grotius 1583-1645 Harvey 1578-1651 Hobbes 1588-1679 Kepler 1571-1630 Monteverdi 1567-1643 Poisson 1584-1665 Rubins 1577-1640	Ampere 1775-1836 Beethoven 1770-1827 Dalton 1766-1844 Fourier 1768-1830 Gauss 1777-1855 Hegel 1770-1831 Ingre 1780-1851 LaPlace 1749-1827 Malthus 1766-1834 Oersted 1777-1858 Owen 1771-1858 Rumsford 1753-1814 Young 1773-1829	Bell 1847-1922 Boltzman 1844-1906 Cezanne 1839-1906 Eastman 1854-1932 Edison 1847-1913 Gibbs 1839-1903 Mach 1838-1916 Neitzsche 1844-1900 Peirce 1839-1914 Zola 1840-1902	Bourbaki fl.1938 Calder 1898-1976 Chadwick 1891-1974 Dirac 1902-1984 Fermi 1901-1954 Godel 1906-1978 Keynes 1883-1946 Vygotsky 1896-1934 Wertheimer 1880-1943 Weyl 1885-1955 Wilder 1897-1975 Wittgenstein 1889-1951 Yukawa 1907-1981	
					c. 470	c. 250	c. 1180	c. 1420	c. 1630	c. 1830	c. 1885	1948	
WHOLES	Relations	domestication early towns kinship systems Stonehenge trading Trobriand trade	10 Commandments c. 1100 alphabetic writing c. 1000 large scale trading c. 1000 Moses c. 1200		Empedocles c. 494-434 Severe Style c. 490-450 The wedding of c. 470	Apollonius 262-200 Archimedes 287-212 Aristarchus c. 310-230 Jesus 4 BC-29 AD Eratosthenes 276-195 Horace 65-8 Livy 59-17 AD Lucretius 96-52 Ovid 43-18 AD Pantheon 25 AD Virgil 70-19	Leonardo of Pisa 1175-1250 Magna Carta 1215 Maimonides 1135-1204 St. Dominic 1170-1221 St. Francis of Assisi 1182-1226 University of Paris c. 1200	Brunelleshi 1379-1446 Donatello 1386-1446 Fra Angelico 1387-1458 Gutenburg c. 1395-1468 Joan of Arc 1410-1431 Master-Flemalle fl.1425-28 Van Eyck 1385-1441	Bernini 1598-1680 Cromwell 1599-1658 Desargues 1593-1662 Descartes 1596-1650 Fermat 1601-1665 Poussin c. 1593-1665 Torricelli 1608-1648	Berlioz 1803-1869 Carnot 1796-1832 Compte 1798-1857 Delacroix 1798-1863 Faraday 1791-1867 Galois 1811-1832 Gericult 1791-1824 Hamilton 1805-1865 Hugo 1802-1885 Kierkegaard 1813-1855 Schopenhauer 1788-1860 Schubert 1797-1828 Tocqueville 1805-1859	Cantor 1845-1918 Debussey 1862-1918 Gauguin 1848-1903 Gibbs 1839-1903 H. James 1843-1916 Hertz 1857-1894 Hilbert 1862-1943 Lautrec 1864-1901 Michelson 1852-1931 Seurat 1859-1891 Sullivan 1858-1917 W. James 1842-1910	Chomsky 1928- Crick 1916- Erikson 1902-1994 Feynman 1928-1992 Levi-Strauss 1908-1987 Orwell 1903-1950 Pasternak 1890-1960 Pauling 1907-1994 Pollock 1912-1956 Skinner 1904-1989 Von-Neumann 1903-1957 Watson 1928- Weiner 1894-1964	
					c. 460	c. 150	c. 1230	c. 1480	c. 1650	1848	c. 1890	1963	
WHOLES	Transformations	breeding early bronze metal working pictographs wheel pottery	coinage c. 700 Hebrew Prophets fl.700 Homer fl.750		Anaxagoras c. 500-428 Herodotus 485-425 Sophocles 496-406 Zeno 495-435	Arch c. 312 Augustine 354-430 Column of Trajan 106-113 Diophantus c. 250 Galen c. 130-201 Gospel c. 100 Pappus c. 330 Plotinus 205-270 Ptolemy 100-168 Simplicius c. 525	Chartre Cathedral c. 1194 Frederich II 1198-1250 Grosseteste 1175-1253 Henry II 1154-1189	Bosch 1450-1516 Bottocelli 1445-1510 Castagno 1423-1457 della Francesco 1410-1492 Lorenzo DiMedici 1449-1492 Savonarola 1452-1498	Boyle 1627-1691 Colbert 1619-1683 Hooke 1635-1703 Milton 1608-1674 Pascal 1623-1662 Rembrandt 1606-1669 Spinoza 1632-1677 Vermeer 1632-1675	Boole 1815-1864 Bronte 1818-1848 Clausius 1822-1888 Courbet 1819-1877 Dickens 1812-1870 Joule 1818-1889 Kelvin 1824-1907 Marx 1818-1883 Mayer 1814-1878 Melville 1819-1899 Riemann 1826-1866 Thoreau 1817-1862	Becqueral 1832-1908 Checkov 1860-1904 Conrad 1857-1924 Durkheim 1858-1917 Kipling 1865-1936 Lie 1842-1909 Lorentz 1853-1928 Poincare 1854-1912 Roentgen 1845-1923 Thomson 1856-1940 Van Gogh 1853-1890 Wilde 1854-1900	Doctorow 1931- Ehrlich 1932- Gell-Mann 1929- Gould 1941-2002 Guth 1947- Hawking 1942- Kuhn 1922-1997 Michener 1907-1997 Piaget 1896-1980 Solzhenitsyn 1918-1996 Vonnegut 1922-2007 Whole Earth Cat 1968	

Most of the people or works represented here are what I believe are the most important as well as good representatives of the phase of knowledge. I have chosen to not focus on knowledge developed mainly by Western civilizations because it would complicate the purpose of this chart which is to enable you to get a good sense of the evolution of knowledge and to focus on well defined times. Asian and Arab people and the same early phases and follow the same sequence, but at different times and different rates. I hope you and others will both extend this pattern to other places and groups and add additional works and people to enrich the phases. I suggest you also compare similar phases as different times to play with the pattern. Finally, it is generally very hard to clearly recognize the revolutionary making phase boundaries while you are in them. Such is the case for me with Artifacts, so please take the people and dates i have put down as simply markers for a fundamental change in the works we create.

Open-Web Testing



Most people would credit the microprocessor as the innovation that began and continues to drive the digital age. No doubt it makes our new age possible, but I would rather give the credit to an idea and not an object. In 1965 Gordon Moore, who would soon become a founder of Intel, was asked to predict the future of semiconductors. He forecast that the number of components (effectively the number of transistors) would double every year. Ten years later, at the beginning of the microprocessor era, he prophesied they would double every 2 years. This prediction, known as “Moore’s law,” I believe opened the floodgates of innovation to the technology of the digital age. He made it possible to imagine the future and to dream up the tools of that future.

In a similar vein, I suggest an idea to transform education. **What if we made all significant tests Open-Web?** What if we make a minor change in the instructions in the Common Core tests, the SAT’s, the AP’s and of course teacher generated tests, by taking out the word “**not**”? What if they said that students may **use** any Web-connected device when they take the exam? Yes, they could email and text, use Google and Wikipedia, and even Skype with a friend or mentor. Their only limits would be those reasonable ones dealing with pornography, safety, or racism/sexism.

The business world we seek to prepare our kids for, we would never imagine solving a real problem by bringing a group of folks together in a closed room and telling them to turn off their cell phones, close their computers, not talk to each other, and take an hour and a sheet of paper to figure out the solution. If we are serious about creating our next generation of problem solvers with 4Cs skills, why would we possibly imagine evaluating them based on testing traditional manufacturing repetitive skills?

Of course, this would mean that, assessment would have to change in profound ways. Because factual questions or derivations could be just looked up, I cannot imagine paper tests in the 21st century any more than I can imagine paper solutions to STEM, corporate, or governance problems. Tests measuring spelling, definition, computation, translation, regurgitation, and the solving of standardized problems would be rendered meaningless. As we have seen, evaluation methodology in the 21st century must surely include some form of portfolio or juried project. It likely will also include attitudinal measures, for students who are excited, confident, and concentrating are going to be learning. There are other measurement tools in this digital age like simulations, coding web pages, and performing virtual and real experiments. We can look to the business world for evaluation tools and methodologies, to the sports world, as well as to a rich variety of methods already available to us in the education world. We need to learn to think in terms of digital age problem solving and technologies rather than cheap but now meaningless paper questions.

In many respects this shift has already been occurring. As schools sought simple quantitative automated measures that could be easily digitized, they chose SAT scores. But like all such mechanical assessments, SAT scores have inflated by a 100+ points over the past 20 years as high school students started doing serious test prep. Today, SAT scores are virtually meaningless for the upper quartile of high school students and more and more colleges are turning to AP test scores, particularly AP Calculus test scores as their external evaluation of student capability. But these tests are also proving poor predictors of college success as test prep replaces real long-term learning as the goal of many high school calculus courses. Instead of preparing students for the past, schools would be driven to prepare students for the future if that is what we began to measure.

> *We can make it real, transforming education, opening the floodgates to innovation by simply taking the "not" out of the instructions on those tests. Change "You are not allowed to use any Internet connected devices on this test." To "You ARE allowed to use any Internet connected device on this test.*

Open-Web Schools

Cambridge MA in an earlier time

My city has well-earned its nickname "The People's Republic of Cambridge" as a progressive city that involves and invokes citizen input and preaches liberal values. But Cambridge is far more interesting and complex than the moniker would lead one to expect. One of the oldest settlements in the United States, founded in 1632, this city of 100,000 has a long history of being a leading industrial center manufacturing glass, brick, ropes and cables, books, telescopes, shoes, furniture, iron lungs, candy, yes candy. Today, we primarily manufacture knowledge in technology, education, and pharmaceuticals.

Thanks to those new manufacturers and to a robust tourist trade, we are a very rich city where real estate is at a premium and services are highly regarded. We are also a great education city with the primary campuses of 3 major universities[124] within our city boundaries and next-door

[124] Harvard University, Massachusetts Institute of Technology, Lesley University

neighbors to Tufts, Boston University, and more. And we have long been the site of invention taking substantial credit for things like the telephone, instant photography, and spreadsheets.

Yet, in many ways, we are a deeply conservative city that faces the same kinds of issues our nation faces. Though we are the seat of education, our schools are not considered exemplary, and if we look at their structure and curriculum, despite many citizen-driven reform movements over the past 50 years, they are quite traditional in form and substance. Our school population, hovering near 7,000 students is split close to 50-50 in the free and reduced lunch category, has many immigrant students and their families speaking some 30+ different languages.

Though by and large we have had visionary school committees and superintendents who in 2017 were spending over $27,000 per pupil, 1st in the state, a rash of new or remodeled buildings, and the strong support of the community, we ranked in that same year 61st[125]. What concerns me more is that less than 40% of our high school graduates succeed in getting a bachelor's degree within 6 years.[126] My deep contact with this educational community over nearly half a century and these painful results have been convincing evidence that our current model of education is truly overly-optimized and incapable of producing the results we, like every other community, must commit to. As my friend, city councilor Craig Kelley speaking about Cambridge and the United States as a whole, told me recently, " Whatever we have been doing for years has had no meaningful impact."

We are also a deeply conservative city in the way we treat technology in our schools. We are closed-web. Most websites including YouTube are blocked by a program called Barracuda. Get on a school computer and go to YouTube or most other websites on the Internet and a nasty black screen pops up saying something like "Barracuda has barred you from going to this site." Those sites students are allowed to go to belong mainly to major publishers who have the legal and engineering staff to claim that they have super protected student data and whose content the schools pay for. Sites and apps that teachers seek to make available to the classes have to be applied for from the IT department, sometimes a daunting process, and most of the publishers of those sites that cannot or will not sign the city's stringent data policy then require parents to opt in. Teachers who use their school machines are told that their Web activities, though not blocked, are all tracked. So, despite having a hardware rich environment, technology use is very limited and controlled.

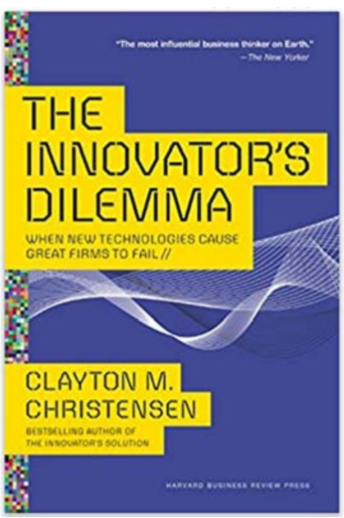
This policy, which I am told repeatedly is demanded by the city's legal department, is not an educational policy. It is policing. It is required, they say, to stave off lawsuits. Indeed, most of our

[125] Boston Magazine, "Best Public Schools in 2017"
https://www.bostonmagazine.com/best-school-districts-boston-2017-chart/
[126] Cambridge Rindge and Latin class of 2006 6-year college graduation rate study.

students can get on most any website they want, either in school or out of school, by using their cell phones or VPN access. Try as we might to protect our kids from things they should not see and from data thieves who would steal from them or molest them; we are not doing so. We have forgotten that our mission is education and not policing. We do not protect our kids when we do not teach them to protect themselves, when we try to build a moat around our schools that they are constantly swimming across anyway.

These policies would not be so destructive if they did not inhibit digital age learning. Our students and teachers need only look out the windows of our schools to see the businesses we should be preparing our kids for. By closing off the Web, by inhibiting technology, we are forcing our administrators, teachers, and students to work in a 19th century educational system without creativity, critical thinking, collaboration, or communication. We suffocate the creativity of teachers and students by withholding the tools and the information they need to learn to use. And we have failed to educate them to protect themselves from the real predators of the digital age. If we are to double our college success rates for our Cambridge kids, we must make these creative tools available to our teachers and kids not just out of school but in school as well. We must become open-web schools.[127]

Make it Change



Open-web schools and open-web testing are big-scale top-down solutions not unlike today's testing mandates,[128] Common Core, or the myriad of large-scale programs that have come and gone through schoolhouse doors. We are prone to look for such solutions to our educational crisis, or as it is so often described, to turn the Titanic.[129] But is the rudder big enough, the captain's command early enough, or the leadership good enough to change course? I neither expect nor hope for state or federal mandates to bring education into the digital age. It is up to each and every one of us to start and drive this process.

I also find it hard to imagine many large or moderately sized school systems willing to experiment on a grand scale or take the reinvention lead, even though they know such transformation is necessary. Nor, can I hope that individual schools, public or private, charter or experimental,

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[127] Now that does not mean that we cannot block those sites which are pornographic, belligerent or dangerous. We can have realistic opt-out rules. And we can make sure we teach our kids how to protect themselves, their families, and their friends. But we must remember that our first mission is education. We do not protect our children by trying to shield them from the real world. They will find it and it will find them no matter how hard we may try to prevent that.
[128] If you want to start turning your schools or classrooms into problem-based-learning spaces, attack the high stakes tests not as an impediment but as problem to be tackled collaboratively.
[129] Or perhaps the Queen Mary.

immediately change their direction. But like today's dynamic entrepreneurial business world, I can easily imagine our schools, teachers, and educational leaders as local innovators begin to reinvent schools. I encourage and advocate this bottoms-up approach. Begin to change your schooling paradigm, begin to practice using technology as your primary educational tool to unite teaching and learning. Begin to think in terms of using technology not just to change *how* but to change *what* our students learn. Begin to challenge your students and your teachers to use real-world tools to solve real-world problems. **To encourage this process, view yourself as an entrepreneur and consider students your customers.**

In the business world VC's ask entrepreneurs to address these questions. I have tried to answer them the spirit of entrepreneurship for education in the digital age. I hope you will do the same.

1. **What is your vision?**

   Use digital technology to prepare students for their future and not our past.

2. **What is your market opportunity?**

   Engage and enable 80% of our kids to get a bachelor's level college degree by using digital technology to make them better learners and learning much less expensive.

3. **What is your product/service?**

   Combine teaching and learning to develop 21st century 4Cs problem solving skills replacing the 3Rs.

4. **What is your competitive advantage?**

   Use digital age real-world tools like spreadsheets, websites, and the Web to develop and share a rich array of interdisciplinary problem-based-learning lessons to provide choice.

5. **Who is your customer?**

   Students of all ages who seek schooling that is relevant, effective, and efficient, preparing them for their life and work

6. **What is your value proposition?**

   What if Math is our STEM 125 lesson exemplar of the future; integrating mathematics, problem-solving, coding, and spreadsheet skills for student-centered-learning.

7. **When faced with a challenge…**

   Ask not what other schools do, ask what the real world would do.

My Thanks

Of the 4Cs of the digital age, none was more important or played a more vital role in my work than Collaboration. This particular work has a long history and many to thank for its ideas and values. I cannot thank you all, and doing so would do little to illuminate the sources of my inspiration. So please forgive me if I have not included so many of you who have been so valuable while I list those few who played a central role in my work life over the past half-dozen years: My partner in *What if Math*, Peter Mili who is a brilliant teacher, great friend, and superb mathematician. My design partner, in the biggest sense of the word, Ryan McQuade who makes learning beautiful. My lunch partner and mentor Steve Bayle who demands answers. My oldest business friend Frank Ferguson who not only reads and edits all my work, but who makes sure that I approach it from the learners side. My muse George Blakeslee who always sends my thinking off in new directions. My brother-in-law Rick Segal who not only supports my work but who asks the right questions about the future of education. My kids, Kori who is always curious and keeps me curious, Brenan whose core focus on problem solving gives me direction, and Arran who said “Dad, write a 2 pager” the single most valuable thing anyone said to me. My wife Betty whose moral compass always guides me. And for the many who deserve thanks for helping me seek to invent the future of education, please subscribe to my blog post for I am likely to tell your story.